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Existence of solutions for some functional integrodifferential equations with nonlocal conditions

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In this paper, we discuss the existence of mild solution of functional integrodifferential equation with nonlocal conditions. To establish this results by using the resolvent operator theory and Sadovskii-Krasnosel'skii type of fixed point theorem and to show the usefulness and the applicability of our results to a broad class of functional integrodifferential equations, an example is given to illustrate the theory.

KEYWORDS
functional differential equations, integro-differential equations, nonlocal conditions, fixed point techniques

MSC CLASSIFICATION
45J05; 34K30; 34K40

1 | INTRODUCTION

For this analysis, to discuss the existence of mild solutions of the following nonlocal functional integrodifferential equation of the form,

$$\begin{aligned} x'(t_1) &= Ax(t_1) + \int_0^{t_1} R(t_1 - s)x(s)ds \\ &+ f\,(t_1, x(t_1), x(b_1(t_1)), x(b_2(t_1)), \ \ldots, x(b_n(t_1))) \ \text{ for } t_1 \in [0, a], \end{aligned} \tag{1.1}$$

$$x(0) = x_0 + g(x). \tag{1.2}$$

Here, A is a linear operator which is closed on Banach space $(X, \|\cdot\|)$ and $[R(t_1)]_{t_1 \geqslant 0}$ be a set of closed linear operator on X and $D(R) \supset D(A)$. The functions f and g are continuous from $[0, a] \times X^{n+1}$ to X and $[0, a]$ to Banach space X, respectively, which are satisfying conditions to be indicated later.

The general form of Equation (1.1) stands the abstract formulation of some functional integrodifferential equations in viscoelasticity, heat flow, and more physical situations see.[1-3] The problem of existence and uniqueness of solutions of the special form as in (1.1) have been discussed by many authors with their distinct techniques. Many authors derived the existence and uniqueness of solutions by using the resolvent operator technique.[4-7] From their results, they gave more important to nonlocal initial condition problems because it has more realistic than the classical one, especially in physical problems, see previous studies.[8-11]

The field of differential calculus has many real-world applications in various fields of science and engineering. The real-time applications and solutions of differential equations are established in other works.[12,13] The researchers started in the recent years to think how to enlarge the solutions of differential equations by constructing operators referred in the literature.[14,15] The existence of solutions for neutral partial differential equation with nonlocal problems was studied by Ezzinbi et al[16,17] and proved the mild solution of (1.1) when $R = 0$ in Fu and Ezzinbi.[18] Then the problem was addressed by Liang et al[19] and many others.[20-22] Recently, Lizama[23] proved the existence of mild solution of the equation of the form (1.1) when g is compact and $R_1(t_1)$ is norm continuous. Recent research focus to use fractional calculus to solve many problems in various fields.[24-29]

In this paper, we approach a new technique, that is, g is not compactness and using iterative methods based on the Sadovskii-Krasnosel'skii type of fixed point theorem technique in Ezzinbi et al[30] and to show that immediate norm continuity of $R_1(t_1)$ associated to (1.1) is to the semigroup $T(t)$ is immediate norm continuity. Motivated by the above mentioned articles,[6,7,16-18,30] the purpose of this paper is to study the existence of solutions for some functional integrodifferential equations with nonlocal conditions (1.1) and (1.2). Throughout, this analysis used the concept of fixed point technique of Sadovskii's type and resolvent operator.

From this analysis, to discuss the following sections. From Section 2, we remember some basic results, definitions, and some lemmas. From Section 3, we discuss the existence of mild solution of the nonlocal problem (1.1) under the given necessary and sufficient for $R_1(t_1)$ associated to nonlocal problem (1.1). From Section 4, we discuss about the functional integrodifferential equations, and the last section, we provide an example to illustrate the theory. Throughout this analysis, used X is Banach space.

2 | PRELIMINARIES

Here, we discuss some definitions and preliminaries. Consider M and N are two Banach spaces and $L(M, N)$ denote the linear bounded operators from M into N and write $L(M)$ when $M = N$. We refer some results on resolvent operator, see also other studies.[4,5,7] Now, we consider the following integrodifferential equation:

$$u'(t_1) = Au(t_1) + \int_0^{t_1} R(t_1 - s)u(s)ds \text{ for } t_1 \geqslant 0,\ u(0) = u_0 \in X. \tag{2.1}$$

Suppose that

- **(H1)** The closed linear operator A is defined on $(X, \|\cdot\|)$ densely. The domain of A has the norm $\|x\| = |Ax| + |x|$ which is also Banach space let it be $(Y, \|\cdot\|)$.
- **(H2)** $[R(t_1)]_{t_1 \geqslant 0}$ be a set of linear operator on Banach space X such that $R(t_1)$ is continuous on Y to X for $t_1 \geq 0$. The function $c : \mathbb{R}^+ \to \mathbb{R}^+$ then $R(t_1)$ is measurable and $|R(t_1)| \leq c(t_1)\,\|y\|\ \forall y \in Y$ and $t_1 \geq 0$.
- **(H3)** For some y in Y, then the map $t_1 \to R(t_1)y \in W^{1,1}_{loc}(\mathbb{R}^+, X)$ and $|\frac{d}{dt_1}R(t_1)y| \leq c(t_1)\,\|y\|\ \forall t_1 \in \mathbb{R}^+$.

Definition 2.1. A linear bounded operator $R_1(t_1)$ on X is a resolvent operator for (2.1) has the following properties are hold:[7]

- (a) $R_1(0) = I$, which is defined on X called identity map and $|R_1(t_1)| \leq Me^{\beta t_1}$ where M, β are constants.
- (b) $R_1(t_1)x$ is strongly continuous $\forall x \in X$ and $t_1 \geq 0$.

(c) $R_1(t_1) \in L(Y)$ for $t_1 \geq 0$. For $x \in Y$, $R_1(\cdot)x \in C^1\left(\mathbb{R}^+, X\right) \cap C\left(\mathbb{R}^+, Y\right)$ and

$$\begin{aligned} R_1'(t_1)x =& AR_1(t_1)x + \int_0^{t_1} R(t_1 - s)R_1(s)xds \\ =& R_1(t_1)Ax + \int_0^{t_1} R_1(t_1 - s)R(s)xds \ \text{ for } \ t_1 \geq 0. \end{aligned}$$

Theorem 2.2. *Suppose that (H1) to (H3) satisfied. Then Equation (2.1) has a resolvent operator if and only if A generates a C_0-semigroup, so that*[5] ***(H4)*** *The infinitesimal generator A generates a C_0-semigroup $(T(t_1))_{t_1 \geqslant 0}$. We need the following definition and lemmas for this paper. The Kuratowski measure of noncompactness is defined by $\gamma(S) = \inf\{d > 0;$ where S is bounded subset of X covered by finite number of sets with diameter $< d\}$.*

Lemma 2.3. *Let two bounded sets $B_1, B_2 \in X$ then*[31]

(1) $\gamma(B_1) = 0$ *if and only if B_1 is relatively compact;*
(2) $\gamma(B_1) = \gamma\left(\overline{B_1}\right) = \gamma\left(\overline{co}B_1\right)$, *where $\overline{co}B_1$ is the closed convex hull of S_1;*
(3) $\gamma(B_1) \leq \gamma(B_2)$ *when $B_1 \subset B_2$;*
(4) $\gamma(B_1 + B_2) \leq \gamma(B_1) + \gamma(B_2)$;
(5) $\gamma(B_1 \cup B_2) \leq \max\{\gamma(B_1), \gamma(B_2)\}$;
(6) $\gamma(B_1(0, r_1)) \leq 2r_1$, *where $B_1(0, r_1) = \{x \in X; |x| \leq r_1\}$.*

From the definition of measure of noncompactness on X, a map $\varphi : B(X) \to \mathbb{R}_+$ satisfies (1)–(5) in above lemma, where $B(X)$ is collection of bounded subsets of X.

Lemma 2.4. *The Lipschitz continuous map $H : X \to X$ defined by $\gamma(H(B)) \leq k\gamma(B)$ for some constant k, where B is a bounded subset of X.*[31]

Lemma 2.5. *The equicontinuous function $G : [0, a] \to X$ and let $x_0 \in [0, a]$ then $\overline{co}(G \cup \{x_0\})$ is also equicontinuous.*[32]

Lemma 2.6. *The bounded set $G \subset C([0, a], X)$ such that $\gamma(G(t_1)) \leq \gamma(G)$ for some $t_1 \in [0, a]$, where $G(t_1) = \{x(t_1); x \in G\}$. Further more if G is equicontinuous on $[0, a]$ and $t_1 \to \gamma(G(t_1))$ is continuous on $[0, a]$ and $\gamma(G) = sup\{\gamma(G(t_1)); t_1 \in [0, a]\}$.*[31]

Definition 2.7. The set of function $G \subseteq L^1([0, a]; X)$ is uniformly integrable if $|g(t_1)| \leq b(t_1)\ \forall g \in G$ and a positive function $b \in L^1([0, a]; \mathbb{R}^+)$.

Lemma 2.8. *If the sequence $\{x_n\}_{n\in\mathbb{N}}$ contained in $L^1([0, a]; X)$ is integrable uniformly, then $t_1 \to \gamma(\{x_n\}_{n\in\mathbb{N}})$ is measurable $\forall n \in \mathbb{N}$ and*[33]

$$\gamma\left(\left\{\int_0^{t_1} x_n(s)ds\right\}_{n=1}^{\infty}\right) \leq 2\int_0^{t_1} \gamma\left(\{x_n(s)\}_{n=1}^{\infty}\right) ds.$$

Lemma 2.9. *The subset G of X is bounded and $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in G. For each $\epsilon > 0$ such that*[34]

$$\gamma(G) \leq 2\gamma\left(\{x_n\}_{n=1}^{\infty}\right) + \epsilon.$$

Lemma 2.10. *Let $0 < \epsilon < 1$, $h > 0$ and denote $C_n^m = \binom{m}{n}$ for all $0 \leq m \leq n$ such that*[35]

$$S_n = \epsilon^n + C_n^1 \epsilon^{n-1}\frac{h^1}{1!} + C_n^2 \epsilon^{n-2}\frac{h^2}{2!} + \ \dots . + \frac{h^n}{n!}, n \in \mathbb{N}.$$

Then $\lim_{n\to\infty} S_n = 0$.

Definition 2.11. Let K be a convex closed subset of X. Let $M, S : K \to X$ be nonlinear functions. For $x_0 \in X$ and N be subset of K then defined by

$$F^{(1,x_0)}(M,S,N) = \{x \in K; x = Sx + My \text{ for any } y \in N\} \text{ and}$$
$$F^{(n,x_0)}(M,S,N) = F^{(1,x_0)}\left(M,S,\overline{co}\left(F^{(n-1,x_0)}(M,S,N) \cup \{x_0\}\right)\right) \text{ for } n > 1.$$

Definition 2.12. Let K be a convex closed subset of X and φ is measure of noncompactness on X.[32] Let two bounded mappings $M, S : K \to X$ and $x_0 \in K$, then we call M is S-convex power condensing operator about x_0 and n_0 with respect to φ if some bounded set $N \subseteq K$ and $\varphi(N) > 0$ then $\varphi\left(F^{(n_0,x_0)}(M,S,N)\right) < \varphi(N)$.

The following fixed point technique is used to investigate our analysis.

Theorem 2.13. *Let K be a bounded convex closed subset of X and φ is a measure of noncompactness on X.*[30]

Define $M, S : K \to X$ are two continuous mappings satisfies

(a) *S is strict contradiction,*
(b) *$Sx + My \in K$, $\forall x, y \in K$,*
(c) *For $x_0 \in X$, there exist n such that M is S-power convex condensing with respect to φ.*

Then $S + M$ has at least one fixed point in K.

3 | IMPORTANT RESULTS

Here, we prove the immediate norm continuity of $R_1(t_1)$ connected to (2.1) is the immediate norm continuity of $T(t_1)$ generated by A. We need the following lemma for our analysis.

Lemma 3.1. *Suppose (H_1) to (H_4) are hold and $(T(t_1))_{t_1 \geqslant 0}$ is strongly continuous semigroup generated by A.*[5] *Let $(R_1(t_1))_{t_1 \geqslant 0}$ be the resolvent operator of (2.1). Then*

$$R_1(t_1)x = T(t_1) + \int_0^{t_1} T(t_1 - s)Q(s)xds, \tag{3.1}$$

$$T(t_1)x = R_1(t_1)x + \int_0^{t_1} R_1(t_1 - s)W(s)xds, \tag{3.2}$$

with

$$Q(s)x = R(0)\int_0^s R_1(u)xdu + \int_0^s R'(s-u)\int_0^u R_1(v)xdvdu$$

and

$$W(s)x = -R(0)\int_0^s T(u)xdu - \int_0^s R'(s-u)\int_0^u T(v)xdvdu,$$

where $Q(\cdot)$, $W(\cdot)$ are bounded operators, there exists $\kappa(a)$ and $\kappa'(a)$ so that $|Q(s_1)| \leq \kappa(a)$ and $|W(s_1)| \leq \kappa'(a)$ for $0 \leq s_1 \leq a$ and $Q(\cdot)x$, $W(\cdot)x \in C([0,\infty), X)$, $\forall x \in X$.

Theorem 3.2. *Suppose that (H_1) to (H_4) are hold. Then $(T(t_1))_{t_1 \geqslant 0}$ is norm continuous for $t_1 > 0$ if and only if $(R_1(t_1))_{t_1 \geqslant 0}$ corresponding (2.1) is norm continuous for $t_1 > 0$.*

Mild solution: The mild solution of (1.1) is a continuous function x: $[0,a] \to X$ satisfying

$$\begin{aligned} x(t_1) =& R_1(t_1)(x_0 + g(x)) \\ &+ \int_0^{t_1} R_1(t_1 - s)f\,(s, x(s), x(b_1(s)), x(b_2(s)), .., x(b_n(s))) \text{ for } t_1 \in [0,a]. \end{aligned} \tag{3.3}$$

To ensure that the following assumptions.

(H5) The function $f : [0,a] \times X^{n+1} \to X$ satisfied Caratheodory properties; ie, $f(\cdot, x)$ is measurable $\forall x \in X^{n+1}$ and $f(t_1, \cdot)$ is continuous to each $t_1 \in [0,a]$.
(H6) The semigroup $(T(t_1))_{t_1 \geqslant 0}$ is continuous with norm for $t_1 > 0$.
(H7) There exists $\rho_k \in L^1([0,a]; \mathbb{R}^+)$ and for each positive number $k \in N$ such that

$$\sup_{\|x_0\|,\|x_1\|,\ldots,\|x_n\| \le k} \|f(t_1, x_0, x_1, \ldots, x_n)\| \le \rho_k(t_1) and$$
$$\liminf_{k\to\infty} \frac{1}{k} \int_0^a \rho_k(s)ds = r_1 < \infty.$$

(H8) There is a positive number r_g implies that

$$\|g(x) - g(y)\| \le r_g \|x - y\| \quad \text{for } x, y \in C([0,a]; X).$$

(H9) There is a function $C \in L^1\left([0,a]; \mathbb{R}^+\right)$ and some bounded set $E \subseteq X$ implies that

$$\gamma(f(t_1, E)) \le C(t_1)\gamma(E).$$

Next, we discuss our existence result.

Theorem 3.3. *Assume that* (H1) *to* (H9) *are hold. Then (1.1) has at least one mild solution on* $[0,a]$ *if*

$$L_a\left(r_g + r_1\right) < 1, \tag{3.4}$$

where $L_a = \sup_{0 \le t_1 \le a} \|R_1(t_1)\|$.

Proof. Define $S, M : C([0,a]; X) \to ([0,a]; X)$ by

$$(Sx)(t_1) = R_1(t_1)(x_0 + g(x)) \quad \text{for } t_1 \in [0,a],$$
$$(My)(t_1) = \int_0^{t_1} R_1(t_1 - s) f(s, y(s), y(b_1(s)), \ldots, y(b_n(s)))\, ds \quad \text{for } t_1 \in [0,a].$$

Then $x(t_1)$ is mild solution of (1.1) if $x(t_1)$ is a fixed point of $S+M$. First to prove the continuity of M, S on $C([0,a]; X)$.

Consider the sequence $(x_n)_n$ in $C([0,a]; X)$, then $\lim_{x\to\infty} x_n \to x$ for any $x \in [0,a]$. By assumptions (H_5) for $s \in [0,a]$, then
$\lim_{n\to\infty} f(s, x_n(s), x_n(b_1(s)), \ldots, x_n(b_n(s))) = f(s, x(s), x(b_1(s)), .., x(b_n(s)))$.

$$\begin{aligned}
\|Mx_n - Mx\| = &\left\| \int_0^{t_1} R_1(t_1 - s) f(s, x_n(s), x_n(b_1(s)), .., x_n(b_n(s)))\, ds \right.\\
&\left. - \int_0^{t_1} R_1(t_1 - s) f(s, x(s), x(b_1(s)), .., x(b_n(s)))\, ds \right\|\\
&\le L_a \int_0^a \| f(s, x_n(s), x_n(b_1(s)), \ldots, x_n(b_n(s)))\\
&- f(s, x(s), x(b_1(s)), .., x(b_n(s))) \| ds.
\end{aligned}$$

□

By using dominated convergence theorem, we have $\lim_{n\to\infty} \|Mx_n - Mx\| = 0$. Hence, M is continuous. Next, we prove S is continuous. Now,

$$\|Sx - Sy\| \le \|R_1(t_1)\| \|g(x) - g(y)\| \le L_a r_g \|x - y\|. \tag{3.5}$$

Hence, S is continuous. For $k > 0$ and $B_k = \left\{x \in C([0,a];X^{n+1}) : \|x\| \le k\right\}$. Let $x, y \in B_k$ and $t_1 \in [0,a]$, then

$$\begin{aligned}\|(Sx)(t_1) + (My)(t_1)\| =& \|R_1(t_1)(x_0 + g(x)) \\ &+ \int_0^{t_1} R_1(t_1 - s) f\,(s, y(s), y(b_1(s)), \ldots, y(b_n(s)))\,ds\| \\ \le& L_a(\|x_0\| + \|g(0)\| + kr_g) + L_a \int_0^a \rho_k(s)ds.\end{aligned} \tag{3.6}$$

To prove, for some $k > 0$ such that $Sx + My \in B_k$. If not, for each $k > 0$ and $x, y \in B_k$ such that $Sx + My \notin B_k$, that is, Equation (3.6) becomes

$$k < \|Sx + My\| \le L_a\left(\|x_0\| + \|g(0)\| + kr_g\right) + L_a \int_0^a \rho_k(s)ds,$$

dividing k on both sides we obtain

$$1 < \frac{L_a}{k}(\|x_0\| + \|g(0)\|) + L_a r_g + \frac{L_a}{k}\int_0^a \rho_k(s)ds.$$

Taking the lower limit as $k \to +\infty$

$$1 \le L_a\left(r_g + \liminf_{k\to\infty}\frac{1}{k}\int_0^a \rho_k(s)ds\right),$$

using ($H7$), we have

$$1 \le L_a\left(r_g + r_1\right),$$

which is contradiction to our assumption (3.4). Hence, there exists $r_0 > 0$ such that

$$\|(Sx) + (My)\| \le r_0.$$

Thus, $Sx + My \in B_{r_0}\ \forall x, y \in B_{r_0}$. Next to show that S is strict contraction. By ($H8$) and (3.5), we have

$$\|(Sx)(t_1) - (Sy)(t_1)\| \le L_a r_g \|x - y\|,$$

for $x, y \in B_{r_0}$ and for any $t_1 \in [0,a]$. Consequently,

$$\|Sx - Sy\| \le k_1\|x - y\|,$$

where $L_a r_g = k_1 < 1$. So that S is a contraction map.

Finally, we have to prove M is S-power convex condensing. This will be derived following three cases.

Case (i) Let N be a subset of B_{r_0}.

We prove that $M(N) = \left\{Mx = \int_0^{t_1} R_1(t_1 - s) f\,(s, x(s), x(b_1(s)), .., x(b_n(s)))\,ds; x \in N\right\}$ is equicontinuous on $[0,a]$.

Let $t_1 = 0$ and $t_2 > 0$ using ($H7$), then

$$\|Mx(t_2) - Mx(0)\| \le L_a \int_0^{t_2} \rho_k(s)ds,$$

because $\rho_k \in L^1([0,a];\mathbb{R}^+)$ then $\|(Mx)(t_2) - Mx(0)\| \to 0$ as $t_2 \to 0$. Therefore, $M(N)$ is equicontinuous at $t_1 = 0$. Consider $0 < t_1 < t_2 \le a$, we get

$$\begin{aligned}\|(Mx)(t_2) - (Mx)(t_1)\| \le& \int_{t_1}^{t_2} \|R_1(t_2 - s)\|\|f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))\,\|ds \\ &+ \int_0^{t_1} \|R_1(t_2 - s) - R_1(t_1 - s)\|\|f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))\,\|ds.\end{aligned}$$

By Theorem (3.2) and ($H6$), we have

$$\|R_1(t_2-s)-R_1(t_1-s)\| \to 0 \ \text{ as } \ t_2 \to t_1 \forall s \neq t_1.$$

Because $s \to R_1(t_2-s)f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))$ is $L^1([0,a];\mathbb{R}^+)$, using Lebesgue dominated convergence theorem,

$$\int_0^{t_1} \|R_1(t_2-s)-R_1(t_1-s)\|\|f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))\,\|ds \to 0 \ \text{ as } \ t_2 \to t_1.$$

Moreover,

$$\begin{aligned}\int_0^{t_1} \|R_1(t_2-s)-R_1(t_1-s)\|\|f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))\,\|ds \\ \leq \int_0^{t_1} \|R_1(t_2-s)-R_1(t_1-s)\|\rho_k(s)ds.\end{aligned}$$

Hence,

$$\sup_{x\in N} \int_0^{t_1} \|R_1(t_2-s)-R_1(t_1-s)\|\|f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))\,\|ds \to 0 \ \text{ as } \ t_2 \to t_1,$$

we have

$$\|(Mx)(t_2)-(Mx)(t_1)\| \to 0 \text{ as } t_2 \to t_1. \tag{3.7}$$

This implies that $M(N)$ is equicontinuous on $[0,a]$.

Case (ii) Next, we show that the equicontinuity of $F^{(n,0)}(M,S,N)$ on $[0,a]$.

Let $n \geq 1$, let $x \in F^{(1,0)}(M,S,N)$ such that $x = Sx + My$ for some $y \in N$.

For $t_1, t_2 \in [0,a]$, then

$$\begin{aligned}\|x(t_1)-x(t_2)\| =& \|Sx(t_1)+My(t_1)-Sx(t_2)-My(t_2)\| \\ \leq& \frac{1}{1-k_1}\|My(t_1)-My(t_2)\|.\end{aligned}$$

From Equation (3.7), $\|My(t_1)-My(t_2)\| \to 0$ as $t_1 \to t_2$. Thus, $\|x(t_1)-x(t_2)\| \to 0$ as $t_1 \to t_2$, so that $F^{(1,0)}(M,S,N)$ is equicontinuous.

Similarly, $F^{(2,0)}(M,S,N) = F^{(1,0)}\left(M,S,\overline{co}\left(F^{(1,0)}(M,S,N)\cup\{0\}\right)\right)$ is also equicontinuous.

Using induction method, we can prove $F^{(n,0)}(M,S,N)$ is equicontinuous $\forall n \geq 1$.

Case (iii) In this case to prove, there is an integer n_0 such that $\gamma\left(F^{(n_0,0)}(M,S,N)\right) < \gamma(N)$. Let $t_1 \in [0,a]$, we have

$$\begin{aligned}F^{(1,0)}(M,S,N)(t_1) =& \left\{x(t_1), x \in F^{(1,0)}(M,S,N)\right\} \\ \subseteq& \left\{x(t_1)-Sx(t_1), x \in F^{(1,0)}(M,S,N)\right\} + \left\{Sx(t_1), x \in F^{(1,0)}(M,S,N)\right\} \\ \subseteq& \{My(t_1), y \in N\} + \left\{Sx(t_1), x \in F^{(1,0)}(M,S,N)\right\}.\end{aligned}$$

Using measure of noncompactness of S, then

$$\gamma\left(F^{(1,0)}(M,S,N)(t_1)\right) \leq \gamma\left(M(N)(t_1)\right) + k_1\gamma\left(F^{(1,0)}(M,S,N)(t_1)\right).$$

Consequently,

$$\gamma\left(F^{(1,0)}(M,S,N)(t_1)\right) \leq \frac{1}{1-k_1}\gamma\left(M(N)(t_1)\right). \tag{3.8}$$

Because $M(N)$ is bounded, then by Lemma (2.9) for each $\epsilon > 0$ and there is a sequence $\{y_n\}_{n\in\mathbb{N}}$ in $M(N)$ such that

$$\begin{aligned}\gamma\left(M(N)(t_1)\right) \leq& 2\gamma\left(\{y_n(t_1)\}_{n=1}^{\infty}\right) + \epsilon \\ \leq& 2\gamma\left(\left\{\int_0^{t_1} R_1(t_1-s)f\,(s, x(s), x(b_1(s)), .., x(b_n(s)))\,ds\right\}_{n=1}^{\infty}\right) + \epsilon.\end{aligned}$$

Because $\|R_1(t_1-s)f\,(s, x(s), x(b_1(s)), \ldots, x(b_n(s)))\,\| \leq L_a\rho_k(s)$ and $\rho_k \in L^1\left([0,a];\mathbb{R}^+\right)$, by Lemma (2.8) implies that

$$\gamma\left(M(N)(t_1)\right) \le 4L_a \int_0^{t_1} \gamma\left(\{f\left(s, x(s), x(b_1(s)), \ \ldots, x(b_n(s))\right)\}_{n=1}^{\infty}\right) ds + \epsilon.$$

By $(H9)$, we get

$$\begin{aligned}\gamma\left(M(N)(t_1)\right) &\le 4L_a \int_0^{t_1} C(s)\gamma\left(\{x_n(s)\}_{n\in\mathbb{N}}\right) ds + \epsilon.\\ &\le 4L_a\gamma(N)\int_0^{t_1} C(s)ds + \epsilon.\end{aligned} \tag{3.9}$$

Since $C\left([0,a];\mathbb{R}^+\right)$ in $L^1\left([0,a];\mathbb{R}^+\right)$ and $C \in L^1\left([0,a];\mathbb{R}^+\right)$, then for $\delta < \frac{1-k_1}{4L_a}$ there is $\psi \in C\left([0,a];\mathbb{R}^+\right)$ satisfies $\int_0^a |C(s)-\psi(s)|ds < \delta$.

Equation (3.9) becomes

$$\begin{aligned}\gamma\left(M(N)(t_1)\right) \le& 4L_a\gamma(N)\left[\int_0^{t_1} |C(s)-\psi(s)|ds + \int_0^{t_1} |\psi(s)|ds\right] + \epsilon\\ \le& 4L_a\gamma(N)[\delta + \tau t_1] + \epsilon,\end{aligned}$$

where $\tau = \sup_{0\le s\le a}|\psi(s)|$. Letting $\epsilon \to 0$, we get

$$\gamma\left(M(N)(t_1)\right) \le \left(4L_a\delta + 4L_a\tau t_1\right)\gamma(N). \tag{3.10}$$

Using (3.10) in (3.8), we have

$$\gamma\left(F^{(1,0)}(M,S,N)(t_1)\right) \le (\alpha + \beta t_1)\gamma(N), \tag{3.11}$$

where $\alpha = \frac{4L_a\delta}{1-k_1}$, $\beta = \frac{4L_a\tau}{1-k_1}$.

$$\begin{aligned}\text{Now } F^{(2,0)}(M,S,N)(t_1) &\subseteq \left\{x(t_1) - Sx(t_1), x \in F^{(2,0)}(M,S,N)\right\}\\ &\quad + \left\{Sx(t_1), x \in \ F^{(2,0)}(M,S,N)\right\}\\ &\subseteq \left\{My(t_1), y \in \overline{co}\left(\ F^{(1,0)}(M,S,N) \cup \{0\}\right)\right\}\\ &\quad + \left\{Sx(t_1), x \in \ F^{(2,0)}(M,S,N)\right\}.\end{aligned}$$

By Lemmas (2.3) and (2.4), we have

$$\begin{aligned}\gamma\left(\ F^{(2,0)}(M,S,N)(t_1)\right) \le& \gamma\left(M\left(\overline{co}\left(\ F^{(1,0)}(M,S,N) \cup \{0\}\right)\right)(t_1)\right)\\ &+ k_1\gamma\left(F^{(2,0)}(M,S,N)(t_1)\right)\\ \gamma\left(F^{(2,0)}(M,S,N)(t_1)\right) \le& \frac{1}{1-k_1}\gamma\left(M\left(\overline{co}\left(F^{(1,0)}(M,S,N) \cup \{0\}\right)\right)(t_1)\right).\end{aligned} \tag{3.12}$$

By Lemma (2.9), there is a sequence $\{z_n\}_{n\in\mathbb{N}} \subseteq \overline{co}\left(F^{(1,0)}(M,S,N) \cup \{0\}\right)$, such that

$$\begin{aligned}&\gamma\left(M\left(\overline{co}\left(F^{(1,0)}(M,S,N) \cup \{0\}\right)\right)(t_1)\right)\\ &\qquad \le 2\gamma\left(\left\{\int_0^{t_1} R_1(t_1-s)f\left(s, z_n(s), z_n(b_1(s)), .., z_n(b_n(s))\right) ds\right\}_{n=1}^{\infty}\right) + \epsilon\\ &\qquad \le 4L_a \int_0^{t_1} \gamma\left(\{f\left(s, z_n(s), z_n(b_1(s)), .., z_n(b_n(s))\right)\}_{n=1}^{\infty}\right) ds + \epsilon\\ &\qquad \le 4L_a \int_0^{t_1} C(s)\gamma\left(F^{(1,0)}(K,S,N)(s)\right) ds + \epsilon.\end{aligned}$$

Using (3.11), we have

$$\gamma\left(M(\overline{co}\left(F^{(1,0)}(M,S,N)\cup\{0\}\right))(t_1)\right)\leq 4L_a\int_0^{t_1}C(s)(\alpha+\beta s)\gamma(N)ds+\epsilon. \tag{3.13}$$

Using (3.13) in (3.12), then

$$\begin{aligned}\gamma\left(F^{(2,0)}(M,S,N)(t_1)\right)\leq&\frac{1}{1-k_1}\left(4L_a\int_0^{t_1}C(s)(\alpha+\beta s)\gamma(N)ds+\epsilon\right)\\ \leq&\frac{4L_a}{1-k_1}\int_0^{t_1}[|C(s)-\psi(s)|+|\psi(s)|](\alpha+\beta s)\gamma(N)ds+\frac{\epsilon}{1-k_1}\\ \leq&\frac{4L_a}{1-k_1}\left\{(\alpha+\beta t_1)\delta+\tau(\alpha t_1+\frac{\beta t_1^2}{2})\right\}\gamma(N)+\frac{\epsilon}{1-k_1}\\ =&\alpha(\alpha+\beta t_1)+\beta(\alpha t_1+\frac{\beta t_1^2}{2})\gamma(N)+\frac{\epsilon}{1-k_1}\\ =&\left(\alpha^2+2\alpha\beta t_1+\frac{(\beta t_1)^2}{2}\right)\gamma(N)+\frac{\epsilon}{1-k_1}.\end{aligned}$$

Letting $\epsilon\to 0$, we obtain

$$\gamma\left(F^{(2,0)}(M,S,N)(t_1)\right)\leq\left(\alpha^2+2\alpha\beta t_1+\frac{(\beta t_1)^2}{2}\right)\gamma(N).$$

By induction method, we can prove for $n\geq 1$,

$$\begin{aligned}&\gamma\left(F^{(n,0)}(M,S,N)(t_1)\right)\leq(\alpha+\beta t_1)^n\gamma(N)\\ &\gamma\left(F^{(n,0)}(M,S,N)(t_1)\right)\leq\left[\alpha^n+C_n^1\alpha^{n-1}\beta t_1+C_n^2\alpha^{n-2}\frac{(\beta t_1)^2}{2!}+\ldots.+\frac{(\beta t_1)^n}{n!}\right]\gamma(N).\end{aligned} \tag{3.14}$$

By using equicontinuity of $F^{(n,0)}(M,S,N)$, we have

$$\gamma\left(F^{(n,0)}(M,S,N)\right)\leq\left[\alpha^n+C_n^1\alpha^{n-1}\beta a+C_n^2\alpha^{n-2}\frac{(\beta a)^2}{2!}+\ldots.+\frac{(\beta a)^n}{n!}\right]\gamma(N).$$

Because $0<\alpha<1$ and $\beta a>0$ then by Lemma (2.11), for $n_0\in\mathbb{N}$ so that

$$\left[\alpha^{n_0}+C_{n_0}^1\alpha^{n_0-1}\beta a+C_{n_0}^2\alpha^{n_0-2}\frac{(\beta a)^2}{2!}+\ldots.+\frac{(\beta a)^{n_0}}{n_0!}\right]<1.$$

Equation (3.14) becomes

$$\gamma\left(F^{(n_0,0)}(M,S,N)\right)<\gamma(N).$$

Hence, M is S-convex power condensing operator.

By Theorem (2.12), $S+M$ has at least one fixed point in B_{r_0}.

This shows that the problem (1.1) has a mild solution.

4 | GENERAL FUNCTIONAL INTEGRODIFFERENTIAL EQUATION

In the recent years, the functional integrodifferential equations (FIDE) have been developed by many researchers because its general form appeared in various fields such as fluid dynamics, economics, biology, and control theory. The qualitative theory of integrodifferential equations forms a branch of nonlinear analysis. The existence solution of integrodifferential equations has been studied by several researchers, their different views, see previous works.[36-45] Consider the following functional integrodifferential equation:

$$
\begin{aligned}
x'(t_1) = Ax(t_1) + \int_0^{t_1} R(t_1 - s)x(s)ds \\
+ f\left(t_1, x(t_1), \int_0^{t_1} f_1(t_1, s, x(s))ds, \int_0^{t_1} f_2(t_1, s, x(s))ds, .., \int_0^{t_1} f_n(t_1, s, x(s))ds\right)
\end{aligned} \tag{4.1}
$$

along with the Equation (1.2).

Where $A(t_1)$ and $R(t_1)$ are closed linear operator on a Banach space X with dense domain $D(A)$ with independent of t_1, $f_i : I \times I \times X \to X, g : C(I, X) \to X$ and $f : I \times X^{n+1} \to X$ are given functions, here $I = [0, a]$.

Definition 4.1. A continuous function $x(t_1)$ is said to be a mild solution of (4.1) along with (1.2) if

$$
\begin{aligned}
x(t_1) = R_1(t_1)(x_0 + g(x)) + \int_0^{t_1} R_1(t_1 - s) \\
\times f\left(s, x(s), \int_0^s f_1(s, \tau, x(\tau))d\tau, \int_0^s f_2(s, \tau, x(\tau))d\tau, .., \int_0^s f_n(s, \tau, x(\tau))d\tau\right) ds
\end{aligned}
$$

satisfied.

Where $R_1(t_1)$ is a resolvent operator for (4.1). We assume that

(H10) There exists an integrable function $m_i : I \times I \to [0, \infty)$ such that $\|f_i(t_1, s, x)\| \le m_i(t_1, s)\Omega_i(\|x\|)$ for any $t, s \in I, x \in X$ for $i = 1, 2, \ldots, n$. Where $\Omega_i : [0, \infty) \to (0, \infty)$ is continuous nondecreasing function for $i = 1, 2, \ldots, n$

Theorem 4.2. *Assume that all the assumptions of theorem (3.3) except* $(H5)$ *along with* $(H10)$ *are satisfied. Then Equation (4.1) along with (1.2) has at least one mild solution on* $[0, a]$ *provided that*

$$
L_a\left(r_g + \lim_{k\to\infty} \frac{1}{k} \int_0^a \left(\sum_{i=1}^n k + m_i(t_1, s)\Omega_i(k)\right) ds\right) < 1.
$$

Proof. Define the mapping $S, M : C([0, a]; X) \to C([0, a]; X)$ by

$$
\begin{aligned}
(Sx)(t_1) &= R_1(t_1)(x_0 + g(x)) \quad \text{as} \quad t_1 \in [0, a], \\
(My)(t_1) &= \int_0^{t_1} R_1(t_1 - s) \\
&\times f\left(s, y(s), \int_0^s f_1(s, \tau, y(\tau))d\tau, \int_0^s f_2(s, \tau, y(\tau))d\tau, .., \int_0^s f_n(s, \tau, y(\tau))d\tau\right) ds.
\end{aligned}
$$

□

Then $x(t_1)$ is mild solution of (4.1) if $x(t_1)$ is a fixed point for $S+M$. By using the technique in Theorem (3.3), we can easily show that $S + M$ has a fixed point. The proof of this theorem is similar to that of Theorem (3.3), and hence, it is omitted.

5 | APPLICATION

Application 5.1. To use our result to the following partial integrodifferential equation of nonlocal problem,

$$
\begin{aligned}
u'(t, x) &= p(x)u(t, x) + \int_0^t c(t - s)p(x)u(s, x)ds \\
&+ f_1(t)f_2(u(t, x), u(b_1(t), x), u(b_2(t), x), \ldots, u(b_n(t), x)) \text{ for } t \in [0, a] \text{ and } x \in [0, 1], \\
u(t, 0) &= u(t, 1) = 0 \text{ for } t \in [0, a], \\
u(0, x) &= u_0(x) + g(u(t, x)) \text{ for } t \in [0, a] \text{ and } x \in [0, 1].
\end{aligned} \tag{5.1}
$$

Let $X = C_0([0,1];\mathbb{C})$ be the set of continuous functions from [0,1] to $\mathbb{C}$ vanish at 0 and 1 with uniform norm topology. Suppose that

(i) $u_0 \in C_0([0,1];\mathbb{C})$.
(ii) $c : \mathbb{R}_+ \to \mathbb{R}_+$ is C^1 function with $|C^1(t)| \leq c(t)$ for all $t \geq 0$.
(iii) $p : \mathbb{R} \to \mathbb{C}$ is a function with continuous satisfying $\sup_{s\in[0,1]} Re(p(s)) < \infty$ and $\overline{p([0,1])} \cap \{\lambda \in \mathbb{C} : Re\lambda \geqslant c\}$ is bounded for all $c \in \mathbb{R}$.
(iv) $g : \mathbb{C} \to \mathbb{C}$ is continuous function.
(v) Let the integrable function be $f_1 : [0,1] \to \mathbb{R}$ and the Lipschitz function $f_2 : \mathbb{R} \to \mathbb{R}$ with constant Lf_2.

Remark 5.2. Let $f(t,x) = f_1(t)f_2(u(t), u(b_1(t)), u(b_2(t)), .., u(b_n(t)))$, using assumption (v),

$$\begin{aligned}\|f(t,x)\| &\leq \|f_1(t)\|\|f_2(u(t), u(b_1(t)), u(b_2(t)), .., u(b_n(t)))\| \\ &\leq \|f_1(t)\|\left(\|f_2(0)\| + Lf_2\|u\|\right) \leq \|f_1(t)\|\left(\|f_2(0)\| + Lf_2\rho_k\right) \leq \|f_1(t)\|\|Lf_2\rho_k\|,\end{aligned}$$

where $\rho_k = \sup\limits_{\|u_0\|,\|u_1\|,\dots,\|u_n\| \leq k} \|f(t,u_0,u_1,\dots,u_n)\|$. Using Lemma (2.4), we get

$$\gamma\left(f(t,M)\right) \leq |f_1(t)|\gamma\left(f_2(M)\right) \leq |f_1(t)|Lf_2\gamma(M) \quad \text{for each } M \in \mathbb{R}. \tag{5.2}$$

Now, we define the operator A on X as follows:

$$D(A) = \begin{cases} f \in X; p.f \in X \\ Af = p.f. \end{cases}$$

See Engel and Nagel,[46, p121] A generates norm continuous multiplication semigroup $T(t)_{t\geq 0}$ on X given by

$$T(t)f = e^{tp} \quad \text{for } t \geqslant 0 \ and f \in X.$$

To use the abstract of (1.1) for (5.1), we define the operators $R(t) : Y \to X$ as

$$R(t_1)f = c(t_1)A \quad \text{for } t_1 \geqslant 0 \ and f \in D(A).$$

Equation (5.1) can be rewritten as

$$\begin{aligned} x'(t_1) =& Ax(t_1) + \int_0^{t_1} R(t_1 - s)x(s)ds \\ &+ f\left(t_1, x(t_1), x(b_1(t_1)), \dots, x(b_n(t_1))\right) \text{ for } t_1 \in [0,a], x(0) = x_0 + g(x). \end{aligned} \tag{5.3}$$

Clearly, $|R(t_1)y| \leq |c(t_1)Ay| \leq c(t_1)\|y\|$ and

$$|R'(t_1)y| \leq |c'(t_1)Ay| \leq |c'(t_1)|\,\|y\| \leq c(t_1)\|y\| \quad \text{for all } y \in Y \text{ and all } t_1 \in \mathbb{R}_+.$$

Accordingly, the assumptions (H_1) to (H_4) hold. By Theorem (2.2), Equation (5.3) has a resolvent operator $R_1(t_1)_{t\geq 0}$ on X with norm continuous for $t_1 > 0$. Now, assume g is Lipschitz continuous with constant r_g satisfying $L_a\left(r_g + |f_1|Lf_2\right) < 1$, where $L_a = \sup_{0\leq t_1\leq a}\|R_1(t_1)\|$. According to remark (5.2), Theorem (3.3) satisfied. So we have the following result exist.
Proposition 5.3. From the above assumptions the nonlocal problem (5.1) has at least one mild solution on $[0,a]$.

6 | CONCLUSION

In this present work, we study the existence of mild solutions for a class of functional integrodifferential equation with nonlocal conditions. In this paper, we used the condition of nonlocal term g is simple nature that is not compact and the strongly continuous semigroup $T(t)$ is the immediate norm continuity of the resolvent operator assumed by many authors. Also, this result can be obtained from the assumption of A generates a strongly continuous semigroup. Hence, these results can be applied in a similar way to a large class of functional integrodifferential equations of the form (1.1). Next, we discuss the existence and regularity for some partial neutral functional integrodifferential equations in the future work.

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CONFLICT OF INTEREST

This work does not have any conflicts of interest.

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