BHARATHIAR UNIVERSITY, COIMBATORE – 641 046
M. Phil./Ph.DFT/PT - MATHEMATICS

PART I - SYLLABUS
(For the candidates admitted from the academic year 2018-19 onwards)

Paper I - Algebra & Analysis

Paper II - Partial Differential Equations

Paper III - Special Paper (anyone of the following)

1) Nonlinear Dynamics
2) Abstract Control Theory
3) Computational Fluid Dynamics
4) Artificial Neural Systems
5) Fuzzy Sets, Logic and Theory of Neural Networks
6) Hamiltonian Dynamics and Chaos
7) Solid Mechanics
8) Advanced Graph Theory
9) Optimization Techniques
10) Algebraic Topology
11) Operator Theory
12) Fuzzy Sets: Theory and Applications
13) Advanced Topics in Fluid Dynamics
14) General Topology

PAPER I – ALGEBRA & ANALYSIS

Unit I: Modules &The Structure of Semisimple Algebras

Modules: Simple Modules-Semisimple Modules-Structure of Semisimple Modules-Chain conditions-The Radical.

The Structure of Semisimple Algebras: Semisimple Algebras-Minimal Right Ideals-Simple Algebras-Matrices of Homomorphisms- Wedderburn's Structure Theorem- Maschke's Theorem.

Unit II: Radical & Indecomposable Modules

The Radical: The Radical of an Algebra-Nakayama's Lemma-Nilpotent Algebras-TheRadical of a Group Algebra-Ideals in Artinian Algebras.

Indecomposable Modules: Direct Decomposition-Local Algebras-The Krull-Schmidt Theorem.

Unit III: Lebesgue Spaces $L(\Omega)$

Definition and Basic Properties-Completeness of $L_p(\Omega)$-Approximation by Continuous functions-Convolutions and Young's Theorem-Mollifiers and Approximations by Smooth functions-Precompact sets in $L_p(\Omega)$- Uniform Convexity-The Normed Dual of $L_p(\Omega)$- Mixed-Norm L_p Spaces-The Marcinkiewicz Interpolation Theorem.

Unit IV: Sobolev Spaces $W^{m,p}(\Omega)$

Definition and Basic Properties-Duality and Spaces $W^{-m,p'}(\Omega)$- Approximation by Smooth functions on Ω- Approximations by Smooth functions on $\mathbb{R}^n$- Approximation by functions in $C_0^\infty(\Omega)$- Coordinate Transformations.

Unit V: Sobolev Imbedding Theorem

Geometric Properties of Domains- Imbeddings by Potential Arguments- Imbeddings by Averaging- Imbeddings into Lipschitz Spaces- Sobolev's inequality- Variations of Sobolev's inequality- $W^{m,p}(\Omega)$ as a Banach Algebra- Optimality of the Imbedding Theorem- Nonimbedding Theorems for Irregular Domains.

Treatment as in:

R. S. Pierce, **Associative Algebras**, Springer Verlag, New York, 1982.

Unit I: Chapter 2 Sections 2.3-2.7
Chapter 3 Sections 3.1-3.6

Unit II: Chapter 4 Sections 4.1-4.2, 4.6-4.8
Chapter 5 Sections 5.1-5.2, 5.4

R. A. Adams, J. J. F. Fournier, **Sobolev Spaces**, Academic Press, London, 2003. (Second Edition)

Unit III: Chapter 2
Unit IV: Chapter 3
Unit V: Chapter 4

References:

1. T. W. Hungerford, Algebra, Springer- Verlag, New York, 2003.
2. R.A. Adams, Sobolev Spaces, Academic Press, New York, 1975.

PAPER II – PARTIAL DIFFERENTIAL EQUATIONS

Unit I: Second-order Elliptic Equations

Definitions- Existence of weak solutions- Regularity- Maximum principles- Eigenvalues andeigenfunctions- Problems

Unit II: Linear Evolution Equations

Second-order parabolic equations

Unit III: Linear Evolution Equations

Second-order hyperbolic equations- Hyperbolic systems of first-order equations- Semigroup theory- Problems

Unit IV: The Calculus of Variations

Introduction-Existence of minimizers

Unit V: The Calculus of Variations

Regularity- Constraints- Critical points- Problems

Treatment as in:

L. C. Evans, **Partial Differential Equations**, American Mathematical Society, Providence, 1998.

Unit I: Chapter 6, Sections 6.1-6.6
Unit II: Chapter 7, Sections 7.1
Unit III: Chapter 7, Sections 7.2-7.5
Unit IV: Chapter 8, sections 8.1-8.2
Unit V: Chapter 8, Sections 8.3-8.8

References:

1. R. C. McOwen, Partial Differential Equations: Methods and Applications, Second Edition, Pearson Education, New Delhi 2005.

2. M. Renardy and R. C. Rogers, An Introduction to Partial Differential Equations, Springer, New York, 2004.

PAPER III - 1) NONLINEAR DYNAMICS

Unit I: Dynamics of Differential Equations

Integration of linear second order equations - Integration of nonlinear second order equations - Dynamics in the phase plane - Linear Stability analysis - Non autonomous systems.

Unit II: Hamiltonian Dynamics

Lagrangian formulation of Mechanics - Hamiltonian formulation of Mechanics - Canonical transformations - Hamilton-Jacobi equation and action - angle variables integrable Hamiltonians.

Unit III: Classical Perturbation Theory

Elementary perturbation theory - Canonical perturbation theory - Many degrees of freedom and the problem of small divisors - The Kolmogrov- Arnold-Moser theorem.

Unit IV: Nonlinear Evolution Equations and Solitons

Basic properties of the Kdv equation - The inverse Scattering transforms: Basic principles, KdV equation - Other soliton systems - Hamiltonian structure of integrable systems.

Unit V: Analytic Structure of Dynamical Systems

Ordinary differential equations in the complex domain - Integrable systems of ordinary differential equations - Painleve property of partial differential equations.

Treatment as in:

M. Tabor, **Chaos and Integrability in Nonlinear Dynamics**, John Wiley and Sons, New York, 1989.

Unit I	Chapter 1 Sections 1.1 - 1.4,1.6
Unit II	Chapter 2 Sections 2.1 - 2.5
Unit III	Chapter 3 Sections 3.1 - 3.4
Unit IV	Chapter 7 Sections 7.1 -7.6
Unit V	Chapter 8 Sections 8.2 - 8.4

References:

1. M. Lakshmanan, S. Rajasekar, Nonlinear Dynamics, First Edition, Springer-Verlag, New York, 2002.
2. S. H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering (Studies in Nonlinearity), First Edition, Westview Press, USA, 2001.

PAPER III – 2) ABSTRACT CONTROL THEORY

Unit I: Bounded Linear Operators

Uniformly continuoussemigroups of bounded linear operators – Strongly continuous semigroups of bounded linear operators – The Hille-Yosida theorem – Semigroups of Compact operators – Differentiability.

Unit II: Abstract Cauchy Problem

The Homogeneous Initial value problem – The inhomogeneous initial value problem – Regularity of mild solutions for analytic semigroups.

Unit III: Evolution Equations

Evolution systems – Stable families of Generators – An Evolution system in the Hyperbolic case – Regular solutions in the Hyperbolic case – The inhomogeneous equation in hyperbolic case

Unit IV: Nonlinear Evolution Equations

Lipschitz perturbation of linear evolution equations – Semilinear equations with compact semigroups – Semilinear equations with analytic semigroups.

Unit V: Basic Concepts in Control Theory

Controllability, Observability and Exponential Stability

Treatment as in:

1. A. Pazy, **Semigroups of Linear Operators and Applications to Partial Differential Equations**, Springer-Verlag, New York, 1983.
   Unit I : Sections 1.1 - 1.3, 2.3 – 2.4
   Unit II : Sections 4.1, 4.2 and 4.3
   Unit III : Sections 5.1 to 5.5
   Unit IV: Sections 6.1 to 6.3

2. R. F. Curtain and H. Zwart, **Introduction to Infinite Dimensional Linear Systems Theory**, Springer-Verlag, New York, 1995
   Unit V: Sections 4.1 and 5.1

References:

1. G. R. Sell, Y. You, Dynamics of Evolutionary Equations, Springer-Verlag, New York, 2002.

PAPER III –3) COMPUTATIONAL FLUID DYNAMICS

UNIT I: Introduction

Conduction heat transfer – Thermal conductivity – Convection heat transfer – Radiation heat transfer – Dimensions and Units

Governing Equations and Boundary Conditions:

Basics of computational fluid dynamics – Governing equations of fluid dynamics – Continuity, Momemtum and Energy equations – Chemical species transport – Physical boundary conditions – Time-averaged equations for Turbulent Flow – Turbulent–Kinetic Energy Equations – Mathematical behaviour of PDEs on CFD - Elliptic, Parabolic and Hyperbolic equations.

UNIT II: Principles of Convection

Viscous flow- Inviscid flow – Laminar boundary layer on a flat plate – Energy equation of the boundary layer – The thermal boundary layer –Relation between fluid friction and heat transfer – Turbulent Boundary layer – Heat transfer in Laminar tube flow – Turbulent flow in a tube – Heat transfer in High speed flow.

UNIT III: Natural Convection Systems

Free convection heat transfer on a vertical flat plate – Empirical relations for free convection – Free convection from vertical planes, Cylinders, Horizontal Cylinders, Horizontal plates, Inclined surfaces – Nonnewtonian fluids – Simplified equations for Air, Free convection from spheres – Free convection in enclosed spaces – Combined free and forced convection.

UNIT IV: Finite Volume Method for Diffusion

Finite volume formulation for steady state One, Two and Three -dimensional diffusion problems. One dimensional unsteady heat conduction through Explicit, Crank – Nicolson and fully implicit schemes.

UNIT V: Finite Volume Method for Convection Diffusion

Steady one-dimensional convection and diffusion – Central, upwind differencing schemes-Properties of discretization schemes – Conservativeness, Boundedness, Trasnportiveness, Hybrid, Power-law, QUICK Schemes.

Treatment as in:

1. J. P. Holman, **Heat Transfer**, McGraw-Hill, Singapore, 1986.
   Unit I-III
2. H. K. Versteeg and W. Malalasekera, **An Introduction to Computational Fluid Dynamics: The finite volume Method**, Longman, England, 1998.

Unit I & Unit IV-V

References:

1. F.P. Incropera, D. P. Dewitt, T. L. Bergman and A. S. Lavine, Fundamentals of Heat and Mass Transfer, John Wiley & Sons, USA, 2006.

2. S.V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corporation, USA, 2004.

Paper III – 4) ARTIFICIAL NEURAL SYSTEMS

Unit I: Dynamic Neural Units (DNUs)

Nonlinear Models and Dynamics: Models of Dynamic Neural Units (DNUs)- Models and Circuits of Isolated DNUs- Neuron with Excitatory and Inhibitory Dynamics- Neuron with Multiple Nonlinear feedback- Dynamic Temporal behavior of DNN- Nonlinear analysis for DNUs.

Unit II: Continuous-Time Dynamic Neural Networks

Dynamic Neural Networks Structures: An Introduction- Hopfield Dynamic Neural Network (DNN) and its Implementation- Hopfield Dynamic Neural Networks (DNNs) as Gradient-like systems- Modifications of Hopfield Dynamic Neural Networks- Other DNN models- Conditions for Equilibrium points in DNN.

Unit III: Learning and Adaptation in Dynamic Neural Networks

Some observation on Dynamic Neural Filter Behaviors- Temporal Learning Process I (Dynamic Backpropagation)- Temporal Learning Process II (Dynamic Forward Propagation)- Dynamic Backpropagation for Continuous-Time Dynamic Neural Networks.

Unit IV: Stability of Continuous-Time Dynamic Neural Networks

Local Asymptotic Stability- Global Asymptotic Stability of Dynamic Neural Networks- Local Exponential Stability of DNNs- Global Exponential Stability of DNNs.

Unit V: Discrete-Time Dynamic Neural Networks and their Stability

General Class of Discrete-Time Dynamic Neural Networks- Lyapunov Stability of Discrete-Time Nonlinear Systems- Stability conditions for Discrete-Time DNNs- More General Results on Global Asymptotic Stability

Treatment as in:

M. M. Gupta, L. Jin, N. Homma, **Static and Dynamic Neural Networks: From Fundamentals to Advanced Theory**, John Wiley & Sons, Inc. Publications, New Jersey, 2003.

References

1. S. Haykin, Neural Networks: A Comprehensive foundation, Second Edition, Pearson Prentice Hall, New Delhi, 2005.

2. J. M. Zurada, Introduction to Artificial Neural Systems, Jaico Publishing House, Mumbai, 2006.

PAPER III – 5) FUZZY SETS, LOGIC AND THEORY OF NEURAL NETWORKS

UNIT- I : Fuzzy sets and Fuzzy relations

Fuzzy sets – Basic types and basic concepts – Properties of α –cuts – Representations of fuzzy sets – Decomposition Theorems – Extension principle for fuzzy sets, Crisp and fuzzy relations – Projections and cylindric extensions – Binary fuzzy relations – Binary relations on a single set – Fuzzy equivalence relations – Fuzzy compatibility relations – Fuzzy ordering relations – Fuzzy Morphisms – Sup-I compositions of fuzzy relations. Inf-w_i compositions of fuzzy relations.

Unit II: Fuzzy Relation Equations

Introduction – Problem Partitioning – Solution Method – Fuzzy Relations Equations Based on Sup-I Compositions – Fuzzy Relation Equations Based on Inf-w_i Compositions – Approximate Solutions – The Use of Neural Networks.

Unit III: Fuzzy Logic

Introduction – Fuzzy propositions – Fuzzy Quantifiers – Linguistic Hedges – Inference from Conditional Fuzzy Propositions – Inference from Conditional and Qualified Propositions – Inference from Quantified Propositions.

Unit IV: Fuzzy Control

Origin and Objective – Automatic Control – The Fuzzy Controllers, Types of Fuzzy Controllers – The Mamdani Controller – Defuzzification – The Sugeno Controller, Design Parameters – Scaling Factors – Fuzzy Sets – Rules – Adaptive Fuzzy Control – Applications.

Unit V: Neural Network Theory

Neural Dynamics: Activations and Signals – Neurons As Functions – Signal Monotonicity – Biological Activations and Signals – Competitive Neuronal Signals – Neuron Fields – Neuronal Dynamical Systems – Common Signal Functions – Pulse – Coded Signal Functions. Activations Models – Neuronal Dynamical Systems – Additive Bivalent Models – Bivalent Additive BAM – Bidirectional Stability – Lyapunov Functions – Bivalent BAM Theorem.

Text Book for Units I, II & III

Kir G.J and Yuan Bo "Fuzzy sets and fuzzy logic: Theory and applications", Prentice Hall of India, New Delhi, (2002). (Relevant Sections only)

Text Book for Unit IV

Zimmermann H.J., "Fuzzy Set Theory and its Applications", Fourth Edition, Kluwer Academic Publishers, London, (2001). (Relevant Sections only)

Text Book for Unit V

Bart Kosko, "Neural Networks and Fuzzy Systems", Prentice Hall of India, New Delhi, (2001). (Relevant Sections only)

Reference Books:

1. Kaufmann "Introduction to the theory of fuzzy sets", Volume 1 -, Academic Press, Inc., Orlando, Florida, (1973).
2. John N. Moderson and Premchand S. Nair., "Fuzzy Mathematics: An introduction for Engineers and Scientists", -PhysicaVerlag, Heidelberg, Germany, (1998).
3. S. Rajasekaran and G.A. VijayalakshmiPai., "Neural Networks, Fuzzy Logic and Genetic Algorithms Synthesis and Applications". Prentice-Hall of India, New Delhi,(2004).

PAPER III –6) HAMILTONIAN DYNAMICS AND CHAOS

Unit I: The Dynamics of Differential Equations

Integration of linear second order equations - Integration of nonlinear second order equations - Dynamics in the phase plane - Linear Stability analysis.

Unit II: Hamiltonian Dynamics

Lagrangian formulation of Mechanics - Hamiltonian formulation of Mechanics Canonical transformations - Hamilton-Jacobi equation and action - angle variables -integrable Hamiltonians.

Unit III: Classical Perturbation Theory

Elementary perturbation theory - Canonical perturbation theory - Many degrees of freedom and the problem of small divisors - The Kolmogrov- Arnold-Moser theorem.

Unit IV: Chaos in Hamiltonian systems and area-preserving mapping

Area preservingmapping-Fixed points and the poincare-Birkhoff fixed point theorem Homoclinic and heteroclinic points-Criteria for local Chaos.

Unit V: Nonlinear Evolution Equations and Solitons

Basic properties of the Kdv equation - The inverse Scattering transforms: Basic principles, KdV equation - Other soliton systems - Hamiltonian structure of integrable systems.

Treatment as in:

Chaos and Integrability in Nonlinear Dynamics by M.Tabor, **John Wiley and Sons, New York, 1989.**

Unit I Chapter 1 Sections 1.1 - 1.4,
Unit II Chapter 2 Sections 2.1 - 2.5
Unit III Chapter 3 Sections 3.1 - 3.4
Unit IV Chapter 4 Sections 4.2 -4.5
Unit V Chapter 7 Sections 7.2 – 7.6

PAPER III – 7) SOLID MECHANICS

Unit-I: Analysis of Stress

Body Force, Surface Force and Stress Vector -The State of Stress at a Point- Normal and Shear Stress Components - Rectangular Stress Components - Stress Components on an Arbitrary Plane - Equality of Cross Shears- A More General Theorem- Principal Stresses- Stress Invariants - Principal Planes are Orthogonal -The State of Stress Referred to Principal Axes - Mohr's Circles for the Three-Dimensional State of Stress- Mohr's Stress Plane- Planes of Maximum Shear-Octahedral Stresses-The State of Pure Shear- Decomposition into Hydrostatic and Pure Shear States-Cauchy's Stress Quadric- The Plane State of Stress-Differential Equations of Equilibrium- Equilibrium Equations for Plane Stress State- Boundary Conditions - Equations of Equilibrium in Cylindrical Coordinates – Problems

Unit-II: Analysis of Strain

Deformations-Deformation in the Neighborhood of a Point- Change in Length of a Linear Element- Change in Length of a Linear Element-Linear Components- Rectangular Strain Components - The State of Strain at a Point- Change in Direction of a Linear Element-Cubical Dilatation- Change in the Angle between Two Line Elements- Principal Axes of Strain and Principal Strains- Plane State of Strain-The Principal Axes of Strain Remain Orthogonal after Strain- Plane Strains in Polar Coordinates- Compatibility Conditions- Strain Deviator and its Invariants- Problems

Unit-III: 3 Stress-Strain Relations for Linearly Elastic Solids

Generalised Statement of Hooke's Law -Stress-Strain Relations for Isotropic Materials-
Modulus of Rigidity- Bulk Modulus-Young's Modulus and Poisson's Ratio- Relations between the Elastic Constants- Displacement Equations of Equilibrium- Problems

Unit-IV: Energy Methods

Introduction- Hooke's Law and the Principle of Superposition-Corresponding Force and
Displacement or Work-Absorbing-Component of Displacement- Work Done by Forces and Elastic Strain Energy Stored - Reciprocal Relation- Maxwell-Betti-Rayleigh Reciprocal Theorem - Generalised Forces and Displacements-Begg'sDeformeter- First Theorem of Castigliano-Expressions for Strain Energy- Fictitious Load Method-Superposition of Elastic Energies-Statically Indeterminate Structures- Theorem of Virtual Work-Kirchhoff's Theorem-Problems.

Unit-V: Axisymmetric Problems

Thick-Walled Cylinder Subjected to Internal and External Pressures-Lame's Problem-Stresses in Composite Tubes-Shrink Fits- Sphere with Purely Radial Displacements-Stresses Due to Gravitation- Rotating Disks of Uniform Thickness- Disks of Variable Thickness-Rotating Shafts and Cylinders-Problems

Treatment as in

L. S. Srinath, **Advanced Mechanics of Solids**, Third Edition, Tata McGraw Hill Education Private Limited, NewDelhi, 2011.

References:

1. S. M. A. Kazimi, Solid Mechanics, Tata McGraw Hill Education Private Limited, New Delhi, First Revised Edition, 1974.
2. P.S.D. Verma, Theory of Elasticity, Vikas Publishing House, Pvt. Ltd. New Delhi, 1998.

PAPER III – 8) ADVANCED GRAPH THEORY

UNIT I :PLANAR GRAPHS:

Embeddings and Euler's Formula – Drawings in the plane - Dual Graphs-Euler's Formula. Characterization of planar Graphs - Preparation for Kuratowski's Theorem- Convex Embeddings- Planarity Testing.Parameters of Planarity - Coloring of planar Graphs-Crossing Number- Surfaces of Higher Genus.

UNIT II :EDGES AND CYCLES:

Line Graphs and Edge - Coloring – Edge – Colorings – Characterization of Line Graphs. Hamiltonian Cycles – Necessary Conditions – Sufficient Conditions- Cycles in Directed Graphs.Planarity, Coloring, and Cycles – Tait's Theorem – Grinberg's Theorem- Snarks - Flows and cycle Covers.

UNIT III :ADDITIONAL TOPICS:

Matroids- Hereditary Systems and Examples-Properties of Matroids – The Spam function-The Dual of a Matroid – Matroid Minors and planar Graphs – Matroid Intersection-Matroid Union.Ramsey Theory – The pigeonhole principle Revisited- Ramsey's Theorem – Ramsey Numbers- Graph Ramsey Theory- Sperner's lemma and Bandwidth.

UNIT IV :TRIPLE SYSTEMS:

Steiner Triple Systems- λ -Triple systems.

UNIT V:ORTHOGONAL LATIN SQUARES:

Introduction – The Euler and MacNeish Conjecture- Disproof of MacNeish Conjecture - Disproof of the Euler Conjecture - Orthogonal Latin Squares of order $n \equiv 2 (\mathrm{mod} 4)$.

Treatment as in:

1. Douglas B. west , Introduction to Graph Theory ,prentice-Hall,New Jersey,2001.
   UnitI: Chapter 6 – Sections 6.1 to 6.3
   Unit II: Chapter 7 – Sections 7.1 to 7.3
   Unit III: Chapter 8 – Sections 8.1 to 8.3

2. C.C. Lindner, C.A. Rodger, Design Theory, CRC press, New York, 1997.
   Unit IV: Chapter 1 – Sections 1.1 to 1.7
   Chapter 2 – Sections 2.1 to 2.5
   Unit V: Chapter 5 – Sections 5.1 to 5.5

References:

1. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Springer-Verlag, London, 2008.
2. Z.-X.Wan, Design Theory, World Scientific publishing, Singapore,2009.

PAPER III – 9) OPTIMIZATION TECHNIQUES

UNIT I: Dynamic Programming

Elements of the DP Model: The Capital Budgeting - More on the Definition of , the state- Examples of DP models and computations - Problem of Dimensionality in Dynamic programming - Solution of Linear programs by Dynamic programming.

UNIT II: Decision Theory and Games

Decisions under Risk - Decision Trees - Decisions Under Uncertainty - Game Theory.

UNIT III: Inventory Models

The ABC Inventory System - Generalized Inventory Models – Deterministic Models – Just-in-Time (JIT) manufacturing system.

UNIT IV: Queuing Models.

Role of Poisson and Exponential Distribution - Processes Birth and Fousson and Death - Queues with Combined Arrival and Departures - Non-Poisson Queues - Queues with Priorities for Service - Random or Series Queues.

UNIT V: Nonlinear Programming.

Unconstrained Extremal Problems - Constrained Extremal Problems - Nonlinear Programming Algorithm - Unconstrained Nonlinear Algorithms - Constrained Nonlinear Algorithms.

Unit I - Chapter – 10, Unit II - Chapter – 12, Unit – III - Chapter – 14,
Unit – IV - Chapter – 15, Unit V - Chapter – 19, Chapter – 20

Treatment as in:

H. A. Taha, **Operations Research -An Introduction**, Fifth Edition, Prentice Hall of India (P) Limited, New Delhi, 1996.

References:

1. A. Ravindran, D. T. Phillips, J. J. Solberg, Operations Research: Principles and Practice, Second Edition, John Wiley & Sons (Asia), New Delhi, 2006.

2. S. S. Rao, Engineering Optimization, Third Edition, New Age International (p) Ltd, New Delhi, 1996.

PAPER III – 10) ALGEBRAIC TOPOLOGY

Unit I: The Fundamental Group

Homotopy of Paths – The Fundamental Group – Covering Spaces – The Fundamental Group of the Circle – Retraction and Fixed Points

Unit II: The Fundamental Group

The Fundamental Theorem of Algebra – The Borsuk –Ulam Theorem – Deformation Retracts and Homotopy Type – The Fundamental Group of S^n– Fundamental Groups of Some Surfaces

Unit III: Separation Theorem in the Plane

The Jordan Separation Theorem – Invariance of Domain – The Jordan Curve Theorem – Imbedding Graphs in the Plane

Unit IV: The Seifert –van Kampen Theorem

Direct Sums of Abelian Groups **–** Free Products of Groups – Free Groups **–** The Seifert – vanKampen Theorem – The Fundamental Group of a Wedge of Circles

Unit V: Classification of Surfaces

Fundamental Groups of Surfaces – Homology of Surfaces – Cutting and Pasting **–** The Classification Theorem – Constructing Compact Surfaces

Treatment as in:

J. R. Munkres, **Topology**, Second Edition, Pearson Education, New Delhi, 2006.

Unit I: Chapter 9 (Sec 51-55)
Unit II: Chapter 9 (Sec 56-60)
Unit III: Chapter 10 (Sec 61-64)
Unit IV: Chapter 11 (Sec 67-71)
Unit V: Chapter 12 (Sec 74-78)

References:

1. J. Dugundiji, Topology, Allyn and Bacon, Boston, 1966.

2. W. S. Massey, Algebraic Topology- An Introduction, Springer-Verlag , New York, 1976.

PAPER III – 11) OPERATOR THEORY

Unit I: The Kato decomposition property

Hyper-Kernel and Hyper-Range of an operator- Semi-regular operators on Banach spaces- Analytical core of an operator- The Semi-regular spectrum of an operator.

Unit II: The Kato decomposition property

The Generalized Kato decomposition- Semi-Fredholm operators- Quasi-nilpotent of operator- Two-Spectral mapping theorems.

Unit III: The Single-valued Extension Property (SVEP)

Local spectrum and SVEP- The SVEP at a point- A Local spectral mapping theorem.

Unit IV: The Single-valued Extension Property (SVEP) & The SVEP and Fredholm Theory

The Single-valued Extension Property (SVEP): Algebraic spectral subspaces.
The SVEP and Fredholm Theory: Ascent, descent and the SVEP- The SVEP for operators of Kato type.

Unit V: The SVEP and Fredholm Theory

The SVEP on the components of $\rho_k(T)$- The Fredholm, Weyl and Browder spectra – Compressions.

Treatment as in:

P. Aiena, **Fredholm and Local Spectral Theory, with Applications to Multipliers**, Kluwer Academic Publishers, New York, Boston, DorDrecht, London, Moscow, 2004.

Unit I: Chapter 1- Sections 1-4
Unit II: Chapter 1- Sections 5-8
Unit III: Chapter 2- Sections 1-3
Unit IV: Chapter 2- Section 4
Chapter 3- Sections 1-2
Unit V: Chapter 3- Sections 3-5

References:

1. J. B. Conway, A Course in Functional Analysis, Second Edition, Springer- Velag, New York, 1990.

2. K. B. Lawsen, M. M. Neumann, An Introduction to Local Spectral Theory, London Mathematical Society, Monographs 20, Clarendon press, Oxford, 2000.

PAPER III –12) FUZZY SETS: THEORY AND APPLICATIONS

UNIT - I

CRISP SETS AND FUZZY SETS: Introduction –Crisp Sets: An Overview-The Notion of Fuzzy Sets - Classical Logic: An Overview –Fuzzy Logic. OPERATIONS ON FUZZY SETS: General Discussion –Fuzzy Complement-Fuzzy Union –Fuzzy Intersection – Combinations of Operations – General Aggregation Operations. FUZZY MEASURES: Belief and Plausibility measures-Probability measures –possibility and Necessity measures.

UNIT – II

FUZZY SYSTEMS: General Discussion – Fuzzy Controllers: An Overview – Fuzzy Controllers: An Example – Fuzzy Systems and Neural Networks – Fuzzy Automata – Fuzzy Dynamic Systems. PATTERN RECOGNITION: Introduction – Fuzzy clustering-Fuzzy pattern Recognition - Fuzzy Image Processing. APPLICATIONS: General Discussion - Natural,life, and Social Sciences-Engineering –Medicine-Management and Decision Making - Computer Science-Systems Science - Other Applications.

UNIT – III

FUZZY GRAPHS: Introduction – Operations on fuzzy Graphs – Cartesian Product and Composition – Union and Join paths and Connectivity- Bridges and Cut Vertices- Forests and trees- Trees and cycles- A Characterization of Fuzzy Trees –Fuzzy Cut Sets- Fuzzy Chords- Fuzzy Cotrees - Fuzzy Line Graphs-Fuzzy Interral Graphs- Fuzzy Intersection Graphs-The Fulkerson and Gross Characterization-The Gilmore and Hoffman Characterization.

UNIT – IV

INTUITIONISTIC FUZZY SETS: Definition - operations and Relations-Properties - Intuitionistic Fuzzy sets of a Certain Level - Necessity and possibility Operators –Topological Operators-Geometrical Interpretations.

UNIT – V

INTUITIONISTIC FUZZY RELATIONS: Cartesian Products over IFSS – Index Matrix-Basic Definition and properties - Other Definitions and properties - Intuitionistic Fuzzy Index Matrices-Intuitionistic Fuzzy Relations- Intuitionistic Fuzzy Graphs – Example- Experts who order Alternatives –Measurement tools that Evaluate Alternatives- Some Ways of Determining Membership and Non-membership Functions.

Treatment as in:

UNIT I and UNIT II

1. George J. Klir and Bo Yuan, Fuzzy sets and fuzzy logic: Theory and Applications Prentice Hall of India Private Limited. New Delhi,2008.

UNIT III

2.John N. Mordeson and Premchand S. Nair,Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag Heidelberg,2000.

UNIT IV and UNIT V

3. Krassimir T Atanassov, On Intuitionistic Fuzzy Sets Theory, Springer - Verlag, Heidelberg, 1999.

References:

1. A.I. Ban, Intuitionistic Fuzzy Measures: Theory and Applications, Nova Science Publishers, New York, 2006.
2. J.J. Buckley, E. Eslami, An Introduction to Fuzzy Logic and Fuzzy Sets, Physica- verlag, Heidelberg,2002.

PAPER III – 13) ADVANCED TOPICS IN FLUID DYNAMICS

UNIT I:

Some features of viscous flows : Real and ideal fluids – Viscosity - Reynolds number – Laminar and turbulent flows – Asymptotic behavior at large Reynolds number. Boundary layer theory: Boundary layer concepts – Laminar boundary layer on a flat plate – Turbulent boundary layer on a flat plate – Fully developed turbulent flow in a pipe

UNIT II :

Field Equations for flows of Newtonian field : Continuity equation – Momentum equation – Navier Stokes equation – Energy equation – Equation of motion for arbitrary co-ordinate systems – Exact solution of Navier stokes equation – Steady plane flows : Couette – Poiseuille flow – Flow past a circular cylinder – Steady axisymmetric flows – Circular Pipe flow – Flow between two concentric rotating cylinders

UNIT III:

Thermal boundary layers in laminar flow: Derivation of the energy equation - Temperatureincrease through adiabatic compression - Stagnation temperature – Theory of similarity in heat transfer - Exact solutions for the problem of temperature distribution in a viscous flow - Boundary layer simplifications.

UNIT IV:

Magnetohydrodynamics: Electrodynamics of moving media – The electromagnetic effects andthe magnetic Reynolds number - Alfven's theorem – The magnetic energy - The mechanical equations - Basic equations for the incompressible MHD - Steady Laminar motion - Hartmann flow.

UNIT V:

Magneto hydrodynamic waves - waves in an infinite fluid of infinite electrical conductivity -Alfven waves - Magnetohydrodynamic waves in a compressible fluid - Magneto acoustic waves- Slow and Fast waves - Stability - Physical concepts – Linear Pinch-Kink - Sausage and Flute types of instability - Method of small oscillations – Jeans criterion for gravitational stability.

Treatment as in

1. H. Schlichting, K. Gersten, **Boundary - Layer Theory**, Springer-Verlag, New York, 2003, Relevant topics from chapter 1,2,3,5 and 12.
2. V. C. A. Ferraro and C. Plumpton, **An Introduction to Magneto Fluid Dynamics**, Oxford: Clarendon Press, 1966, Relevant topics from chapters 1,2,3 and 5.

References:

1. P. A. Davidson, An Introduction to Magneto hydrodynamics, Cambridge University Press, Cambridge, 2001.
2. P. K. Kundu, I. M. Cohen, Fluid Mechanics, Academic Press, London, 2002.

PAPER III –14) GENERAL TOPOLOGY

Unit -I Identification Topology; Weak Topology

Identification Topology - Subspaces-General Theorems-Spaces with Equivalence Relations - Cones andSuspensions-Attaching of spaces - The Relation K(f) for continuous Maps- weak Topologies.

Unit -II Function Spaces

The Compact -open Topology- Continuity of Composition; The Evaluation Map – CartesianProducts- Application to Identification Topologies-Basis for Z^Y -Compact Subsets of Z^Y- Sequential Convergence in the c-Topology - Metric Topologies; Relation to the c-Topology- Pointwise Convergence-Comparison of Topologies in Z^Y.

Unit -III Homotopy

Homotopy-Homotopy classes-Homotopy and Function Spaces - Relative Homotopy- Retracts andExtendability -Deformation Retraction and Homotopy-Homotopy and Extendability- Applications.

Unit-IV Maps into Spheres and Topology of E^n

Degree of a Map $S^n \to S^n$-Brouwer's Theorem -Further applications of the degree of a Map- Maps of Spheres into S^n- Maps of Spaces into S^n - Borsuk's Antipodal Theorem -Degree and Homotopy-Components of Compact Sets in E^{n+1}-Borsuk's Separation Theorem -Domain Invariance -Deformations ofSubsets of E^{n+1} - The Jordan Curve Theorem.

Unit-V Path Spaces,H-Spaces and Fiber Spaces

Path Spaces - H-Structures - H-Homomorphisms- H-Spaces- Units - Inversion - Associativity -Path Spaces on H-Spaces - Fiber Spaces - Fiber Spaces for the Class of All Spaces- The UniformizationTheorem of Hurewicz- Locally Trivial Fiber Structures.

Text Book:

1. **J.Dugundiji**, Topology, Allyn and Bacon ,Boston ,Twelfth printing 1978.

Unit-I : Chapter VI(page 120-131)
Unit-II : Chapter XII(page 257-274)
Unit-III: Chapter XV(page 315-330)
Unit-IV : Chapter XVI(page 335-361)
Unit-V : Chapter XIX(page 376-404)

Reference Book:

1. **J.R.Munkres,**Topology,Second Edition,Pearson Education, New Delhi,2006.
2. **W.S.Massey,**Algebraic Topology - An Introduction,Springer-Verlag,New York,1976.