Journal of Informatics and Mathematical Sciences
Vol. 9, No. 3, pp. 799–814, 2017
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

Proceedings of the Conference
Current Scenario in Pure and Applied Mathematics
December 22-23, 2016
Kongunadu Arts and Science College (Autonomous)
Coimbatore, Tamil Nadu, India

Research Article

On *RFG*-Closed Sets in Topological Spaces

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Abstract. In this paper, we introduce and study the new class of sets, namely Regular Feebly Generalized Closed (briefly RFG-closed) sets, Regular Feebly Generalized neighborhoods (briefly RFG-nbhd), RFG-interior and RFG-closure in topological spaces and also some properties of new concepts have be en studied.

Keywords. RFG-closed sets; RFG-nighborhoods; RFG-interior and RFG-closure

MSC. 54A02

Received: January 6, 2017 **Accepted:** March 10, 2017

1. Introduction

In 1970, N. Levine [10] introduced the concept and properties of generalized closed (briefly g-closed) sets in topological spaces. Maheswari and Jain [12], Ibraheem [5, 6], Palaniappan and Rao [15] introduced feebly open and feebly closed sets, feebly generalized closed (briefly fg-closed) sets, generalized feebly closed (briefly gf-closed) sets and regular generalized closed sets in topological spaces respectively. In this paper, we define new generalization of closed set called Regular Feebly Generalized closed (briefly RFG-closed) set which lies between closed set and feebly closed sets. We also study some of their properties.

Throughout this paper, X denote the topological space (X,τ) and on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a space (X,τ) the closure of A, interior of A, semi-interior of A, semi closure of A, feebly closure of A, feebly interior of A and the complement of A are denoted by $cl(A)$, $int(A)$, $sint(A)$, $scl(A)$, $fcl(A)$, $fint(A)$ and A^c or X-A, respectively.

Let us recall the following definitions as pre requisites.

2. Preliminaries

Definition 2.1. A subset A of a space (X,τ) is called a

(1) regular open set [19] if $A = int(cl(A))$ and regular closed [19] if $A = cl(int(A))$.

(2) semi-open set [11] if $A \subseteq cl(int(A))$ and semi-closed set [11] if $int(cl(A)) \subseteq A$.

(3) semi-pre open set (= $be\ ta$-open) [1] if $A \subseteq cl(int(cl(A)))$ and semi-pre closed set (= $be\ ta$-closed) [1] if $int(cl(int(A))) \subseteq A$.

(4) δ-closed [21] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi,\ U \in \tau \text{ and } x \in U\}$.

(5) feebly open set [12] if $A \subseteq scl(int(A))$ and feebly closed set [12] if $sint(cl(A)) \subseteq A$.

Definition 2.2. Let X be a topological space and $A \subseteq X$. The intersection of all semi closed (resp. semi-pre closed and feebly closed) subsets of the space X containing A is called the **semi closure [11] (resp. semi-pre closure [1] and feebly closure [12])** of A and denoted by $scl(A)$ (resp. $spcl(A)$ and $fcl(A)$).
It is well known that $scl(A) = A \cup int(clA)$, $spcl(A) = A \cup int(cl(int(A)))$ and $fcl(A) = A \cup sint(cl(A))$.

Definition 2.3. Let X be a topological space and $A \subseteq X$.

(1) The union of all semi open subsets of the space X contained in A is called **semi interior** [11] of A and is denoted by $sint(A)$.

(2) The union of all feebly open subsets of the space X contained in A is called feebly interior [13] of A and is denoted by $fint(A)$.

Definition 2.4. Let X be a topological space. The finite union of regular open sets in X is said to be π**-open set** [9]. The complement of a π-open set is said to be π**-closed set** [9].

Definition 2.5. A subset A of a space (X,τ) is called a

(1) generalized closed (briefly g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(2) weakly generalized closed (briefly wg-closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(3) mildly generalized closed (briefly mildly g-closed) [16] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

(4) regular generalized closed (briefly rg-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(5) regular weakly closed (briefly rw-closed) [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X.

(6) π-generalized closed (briefly πg-closed) [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(7) regular weakly generalized closed (briefly rwg-closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(8) weakly π-generalized closed (briefly $w\pi g$-closed) [17] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.

(9) generalized feebly closed (gf-closed) [6] if $fcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(10) Feebly generalized closed (fg-closed) [5] if $fcl(A) \subseteq U$ whenever $A \subseteq U$ and U is feebly open in X.

(11) Regular Mildly Generalized closed (briefly RMG-closed) set [22], if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular generalized open set in X.

The complements of the above mentioned closed sets are their respective open sets.

3. Regular Feebly Generalized Closed Sets in Topologicalspaces

In this section, we introduce Regular Feebly Generalized Closed sets in topological spaces and obtain some of their basic properties.

Definition 3.1. A subset A of a space (X,τ) is called Regular Feebly Generalized closed (briefly **RFG-closed**) set if $fcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular generalized open (rg-open)set in X. The family of all RFG-closed sets is denoted by $RFGC(X)$.

Theorem 3.2. *Every closed set is RFG-closed set in X.*

Proof. Let A be any closed set in X. Suppose U is rg-open set in X such that $A \subseteq U$. Since A is closed set in X, $cl(A) = A \subseteq U$.
(i.e.) $cl(A) \subseteq U$. But $fcl(A) \subseteq cl(A) \subseteq U$.
(i.e.) $fcl(A) \subseteq U$. Hence A is RFG-closed set. □

The converse of the above theorem need not be true as shown in the below example.

Example 3.3. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then Closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$ and RFG closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Here $\{c\}$ and $\{a,b,d\}$ are RFG-closed sets but not closed sets.

Corollary 3.4. (i) *Every regular-closed set is RFG-closed set in X.*

(ii) *Every δ-closed set is RFG-closed set in X.*

(iii) *Every π-closed set is RFG-closed set in X.*

Proof. (i) Every regular closed set is closed, from Stone [21] and then follows from Theorem3.2.

(ii) Every δ-closed set is closed, from Dontchev and Ganster [9] and then follows from Theorem 3.2.

(iii) Every π-closed set is closed, from Dontchev and Noiri [9] and then follows from Theorem 3.2. □

The converse of Corollary 3.4, need not be true as shown in the below examples.

Example 3.5. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$. Then

(a) RFG closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$.

(b) regular closed sets are X, ϕ, $\{a,d\}$, $\{b,c,d\}$.

(c) δ-closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$.

(d) π-closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{b,c,d\}$.

Now

(i) Let $A = \{a,c,d\}$. A is RFG-closed but not regular closed set in X.

(ii) Let $A = \{a,b,d\}$. A is RFG-closed but not δ-closed set in X.

(iii) Let $A = \{a,c,d\}$. A is RFG-closed but not π-closed set in X.

Theorem 3.6. *Every feebly closed set is RFG-closed set in X.*

Proof. Let A be any feebly closed set in X. Suppose U is rg-open set in X such that $A \subseteq U$. Since A is feebly closed set in X, $fcl(A) = A \subseteq U$ (i.e.) $fcl(A) \subseteq U$. Hence A is RFG-closed set. □

The converse of the above theorem need not be true as shown in the below example.

Example 3.7. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then RFG closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and feebly closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{b,c,d\}$. Let $A = \{a,c,d\}$. A is a RFG-closed set but not feebly closed set.

Theorem 3.8. *Every RFG-closed set is feebly generalized closed set in X.*

Proof. Let A be any RFG-closed set in X. Suppose U is feebly open set in X such that $A \subseteq U$. Since every feebly open set is rg-open set in X, U is rg-open set in X. Since A is RFG-closed set, $fcl(A) \subseteq U$. So, we have $fcl(A) \subseteq U$ whenever $A \subseteq U$ and U is feebly open set in X. Hence A is feebly generalized closed set. □

The converse of the above theorem need not be true as shown in the below example.

Example 3.9. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then RFG closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and fg-closed sets are X, ϕ, $\{d\}$, $\{c,d\}$, $\{a,d\}$, $\{b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Let $A = \{b,d\}$. A is a fg-closed set but not RFG-closed set.

Theorem 3.10. *Every RFG-closed set is generalized feebly closed set in X.*

Proof. Let A be any RFG-closed set in X. Suppose U is open set in X such that $A \subseteq U$. Since every open set is rg-open set in X, U is rg-open set in X. Since A is RFG-closed set, $fcl(A) \subseteq U$. So, we have $fcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X. Hence A is generalized feebly closed set. □

The converse of the above theorem need not be true as shown in the below example.

Example 3.11. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then RFG closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and gf-closed sets are X, ϕ, $\{d\}$, $\{c,d\}$, $\{a,d\}$, $\{b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Let $A = \{c,d\}$. A is gf-closed set but not RFG-closed set.

Theorem 3.12. *Every RFG-closed set is Mildly-g-closed set in X.*

Proof. Let A be any RFG-closed set in X. Suppose U is g-open set in X such that $A \subseteq U$. Since every g-open set is rg-open set in X, U is rg-open set in X. Since A is RFG-closed set, $fcl(A) \subseteq U$. But $cl(int(A)) \subseteq fcl(A) \subseteq U$. (i.e.) $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open set in X. Hence A is Mildly-g-closed set. □

The converse of the above theorem need not be true as shown in the below example.

Example 3.13. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then Mildly-$g$-closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{c,d\}$, $\{a,d\}$, $\{b,d\}$, $\{b,c,d\}$, $\{a,c,d\}$, $\{a,b,d\}$ and RFG closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Here $\{b,d\}$ is a Mildly-g-closed set but not a RFG-closed set.

Note. The definition of Mildly-g-closed sets in X is used in the investigation of wg^*-closed set by O. Ravi *et. al* [18].

Lemma 3.14. (i) *Every wg^*-closed set (mildly-g-closed set) is wg-closed in X* ([18, Theorem 3.4]).

(ii) *Every wg^*-closed set (mildly-g-closed set) is $w\pi g$-closed in X* ([18, Theorem 3.6]).

(iii) *Every wg^*-closed set (mildly-g-closed set) is rwg-closed in X* ([18, Theorem 3.8]).

Corollary 3.15. (i) *Every RFG-closed set is wg-closed set in X.*

(ii) *Every RFG-closed set is $w\pi g$-closed set in X.*

(iii) *Every RFG-closed set is rwg-closed set in X.*

Proof. (i) From Theorem 3.12 and Then follows from Lemma 3.14(i).

(ii) From Theorem 3.12 and Then follows from Lemma 3.14(ii).

(iii) From Theorem 3.12 and Then follows from Lemma 3.14(iii). □

The converse of Corollary 3.15 need not be true as shown in the below examples.

Example 3.16. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Then

(a) RFG closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$.

(b) wg-closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$.

(c) $w\pi g$-closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{b,c\}$, $\{a,c\}$, $\{a,d\}$, $\{c,d\}$, $\{b,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$.

(d) rwg-closed sets are X, ϕ, $\{c\}$, $\{d\}$, $\{a,b\}$, $\{b,c\}$, $\{a,c\}$, $\{b,d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$.

Now

(i) Let $A = \{b,d\}$. A is wg-closed but not RFG-closed set in X.

(ii) Let $A = \{a,c\}$. A is $w\pi g$-closed but not RFG-closed set in X.

(iii) Let $A = \{a,b,c\}$. A is rwg-closed but not RFG-closed set in X.

Theorem 3.17. *Every RFG-closed set is RMG-closed set in X.*

Proof. Let A be any RFG-closed set in X. Suppose U is rg-open set in X such that $A \subseteq U$. As A is RFG-closed set, $fcl(A) \subseteq U$. But (i.e.) $cl(int(A)) \subseteq fcl(A) \subseteq U$. (i.e.) $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is rg-open set in X. Hence A is RMG-closed set. □

The converse of the above theorem need not be true as shown in the below example.

Example 3.18. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then RFG-closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and RMG-closed sets are X, ϕ, $\{b\}$, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{b,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Here $\{b\}$ and $\{c\}$ are RMG-closed sets but not RFG-closed sets.

Remark 3.19. The following examples show that RFG-closed sets are independent of g-closed set (Example 3.20 and Example 3.21), semi-pre closed sets (Example 3.22 and Example 3.23), semi-closed sets (Example 3.24 and Example 3.25), rw-closed sets (Example 3.26 and Example 3.27), πg-closed sets (Example 3.28 and Example 3.29) and rg-closed sets (Example 3.30 and Example 3.21).

Example 3.20. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **g-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Here $\{c\}$ is a RFG-closed set but not a g-closed.

Example 3.21. Let$X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **g-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Here $\{b,d\}$ is a g-closed set but not a RFG-closed set.

Example 3.22. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **semi-pre closed (be ta-closed) sets** in (X,τ) are X, ϕ, $\{a\}$, $\{c\}$, $\{d\}$, $\{a,c\}$, $\{b,c\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$. Here $\{a,b,d\}$ is a RFG-closed set but not a semi-pre closed set.

Example 3.23. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **semi-pre closed (be ta-closed) sets** in (X,τ) are X, ϕ, $\{a\}$, $\{c\}$, $\{d\}$, $\{a,c\}$, $\{b,c\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$. Here $\{a\}$ is a semi-pre closed set but not a RFG-closed set.

Example 3.24. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **semi closed sets** in (X,τ) are X, ϕ, $\{a\}$, $\{c\}$, $\{d\}$, $\{a,c\}$, $\{b,c\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$. Here $\{a,b,d\}$ is a RFG-closed set but not a semi-closed set.

Example 3.25. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **semi closed sets** in (X,τ) are X, ϕ, $\{a\}$, $\{c\}$, $\{d\}$, $\{a,c\}$, $\{b,c\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$. Here $\{a\}$ is a semi-closed set but not a RFG-closed set.

Example 3.26. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and ***rw*-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,b\}$, $\{a,c\}$, $\{b,d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$. Here $\{c\}$ is a RFG-closed set but not a rw-closed set.

Example 3.27. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and ***rw*-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,b\}$, $\{a,c\}$, $\{b,d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$. Here $\{a,b\}$, $\{a,c\}$, $\{b,d\}$ are rw-closed sets but not RFG-closed sets.

Example 3.28. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **πg-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,d\}$, $\{b,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{a,b,d\}$, $\{b,c,d\}$. Here $\{c\}$ is a RFG-closed set but not a πg-closed set.

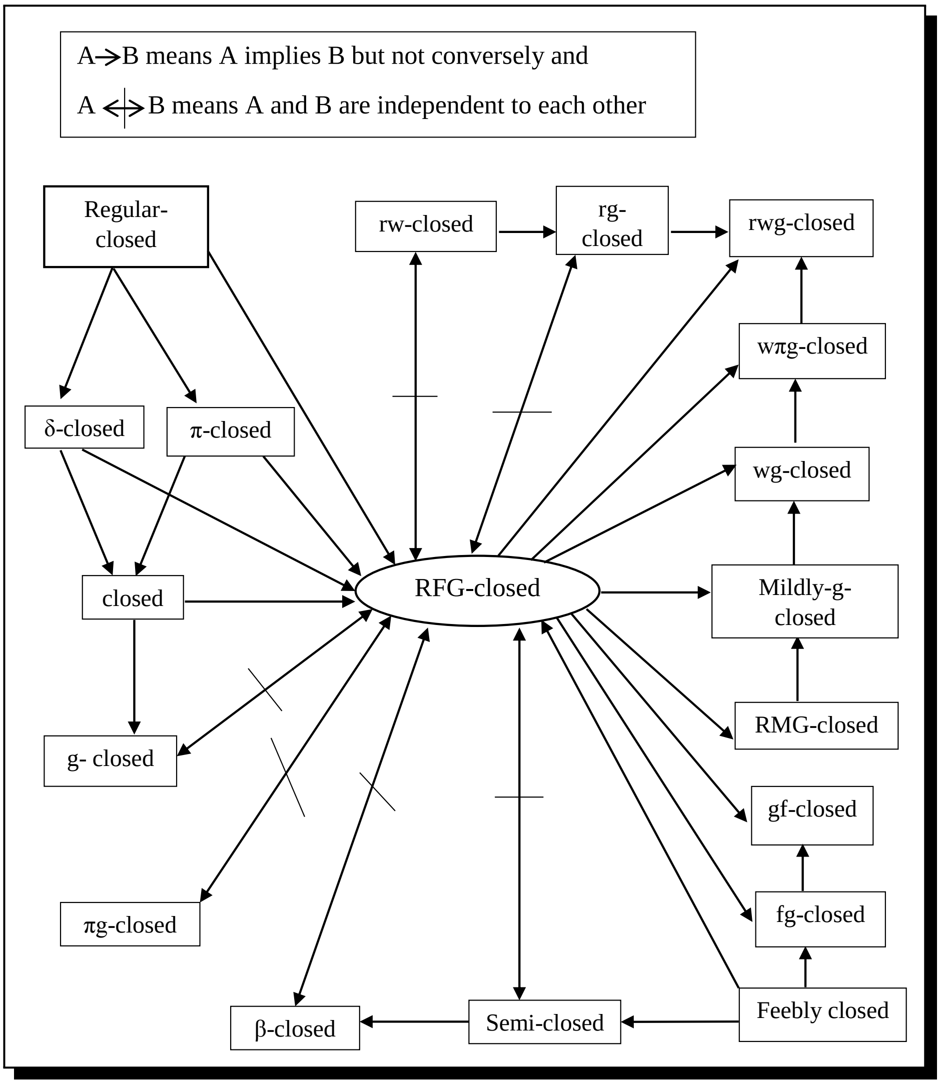
Example 3.29. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and **πg-**

closed sets in (X,τ) are X, ϕ, $\{d\}$, $\{a,d\}$, $\{b,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{a,b,d\}$, $\{b,c,d\}$. Here $\{b,d\}$ is a πg-closed set but not a RFG-closed set.

Example 3.30. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and ***rg*-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{b,c,d\}$. Here $\{c\}$ is a RFG-closed set but not a *rg*-closed set.

Example 3.31. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$ and ***rg*-closed sets** in (X,τ) are X, ϕ, $\{d\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{b,c,d\}$. Here $\{a,b\}$, $\{a,c\}$, $\{b,d\}$, $\{a,b,c\}$ are *rg*-closed sets but not RFG-closed sets.

Remark 3.32. From the above discussions and known results we have the following implications in the following diagram by



Remark 3.33. The intersection of two RFG-closed sets in X need not be RFG-closed set in X. It can be seen by the following example.

Example 3.34. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then RFG closed sets are X, ϕ, $\{d\}$, $\{a,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Now $A = \{a,c,d\}$ and $B = \{b,c,d\}$ are RFG-closed sets in X. Then $A \cap B = \{a,c,d\} \cap \{b,c,d\} = \{c,d\}$ which is not RFG-closed set in X.

Theorem 3.35. *The union of two RFG-closed sets is a RFG-closed set.*

Proof. Let A and B be any RFG-closed sets in X. Let $A \cup B \subseteq U$, U is rg-open. As A and B are RFG-closed sets, $fcl(A) \subseteq U$ and $fcl(B) \subseteq U$. This implies that $fcl(A \cup B) = fcl(A) \cup fcl(B) \subseteq U \Rightarrow fcl(A \cup B) \subseteq U$. Therefore $A \cup B$ is RFG-closed. □

Remark 3.36. The complement of a RFG-closed set need not be RFG-closed set in X. It can be seen by the following example.

Example 3.37. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$. Then **RFG-closed sets** in (X,τ) are X, ϕ, $\{c\}$, $\{d\}$, $\{a,d\}$, $\{c,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}$. Let $A = \{d\}$ is RFG-closed set in X. But $A^c = X - \{d\} = \{a,b,c\}$ is not RFG-closed set.

Theorem 3.38. *Let A be a RFG-closed set of (X,τ) iff $fcl(A) - A$ does not contain any non-empty RFG-closed set.*

Proof. *Necessity Part:* Suppose that A is RFG-closed and let F be a rg-closed set contained in $fcl(A) - A$. Then $A \subseteq X - F$ and $X - F$ is a rg-open set of (X,τ). Since A is RFG-closed, $fcl(A) \subseteq X - F$. This implies $F \subseteq X - fcl(A)$.
Then $F \subseteq (X - fcl(A)) \cap (fcl(A) - A) \subseteq (X - cl(A)) \cap cl(A) = \phi$. Therefore $F = \phi$.

Sufficient Part: Suppose A is a subset of (X,τ) such that $fcl(A) - A$ does not contain any non-empty rg-closed set. Let U be a rg-open set of (X,τ) such that $A \subseteq U$. If $fcl(A) \subseteq U$. Then $fcl(A) \cap U^c$ is a rg-closed set of (X,τ). Hence A is a RFG-closed set. □

Corollary 3.39. *If a subset A of X is RFG-closed set. Then $fcl(A) - A$ does not contain any non-empty regular open set in X.*

Proof. Follows from Theorem 3.38 and the fact that every closed set is rg-closed in X. □

Theorem 3.40. *If A is a RFG-closed set in (X,τ) and $A \subseteq B \subseteq fcl(A)$. Then B is also a RFG-closed set of (X,τ).*

Proof. Let U be a rg-open set in (X,τ) such that $B \subseteq U$. Then $A \subseteq U$. Since A is RFG-closed set, $fcl(A) \subseteq U$. Since $B \subseteq fcl(A)$, $fcl(B) \subseteq fcl(fcl(A)) = fcl(A) \subseteq U$.
(i.e.) $fcl(B) \subseteq U$. Hence B is also a RFG-closed set in X. □

Theorem 3.41. *In a topological space X, if regular generalized open sets of X are $\{X,\phi\}$. Then every subset of X is RFG-closed set.*

Proof. Let X be any topological space and $RGO(X) = \{X,\phi\}$. Suppose A be any arbitrary subset of X, if $A = \phi$. Then X is the only rg-open set containing A and $fcl(A) \subset X$. Hence by Definition 3.1, A is a RFG-closed set in X. □

4. Regular Feebly Generalized Neighborhoods (Briefly RFG-Nbhd)

In this section, we introduce RFG-neighborhood (briefly RFG-nbhd) in topological spaces by using RFG-open sets.

Definition 4.1. A subset A of a space (X,τ) is called Regular Feebly Generalized open (briefly **RFG-open**) set if $X - A$ is RFG-closed set in X. The family of all RFG-open sets is denoted by $RFGO(X)$.

Definition 4.2. (i) Let (X,τ) be a topological space and let $x \in X$. A subset N of X is said to be RFG-neighborhood (briefly, RFG-nbhd) of x if and only if there exists an RFG-open set $G \ni x \in G \subset N$.

(ii) The collection of all RFG-nbhd of $x \in X$ is called RFG-nbhd system at $x \in X$ and shall be denoted by RFG-$N(x)$.

Theorem 4.3. *Every neighborhood N of $x \in X$ is a RFG-nbhd of x.*

Proof. Let N be neighborhood of point $x \in X$. To prove that N is a RFG-nbhd of x. By definition of neighborhood, there exists an open set G such that $x \in G \subset N$. As every open set is RFG-open, G is an RFG-open set in X. Then there exists a RFG-open set G such that $x \in G \subset N$. Hence N is RFG-nbhd of x. □

Remark 4.4. In general, a RFG-nbhd N of $x \in X$ need not be a nbhd of x in X, as seen from the following example.

Example 4.5. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{c,d\},\{a,c,d\}\}$. Then $RFGO(X) = \{X,\phi,\{a\},\{c\},\{d\},\{c,d\},\{a,c,d\}\}$. The set $\{a,d\}$ is RFG-nbhd of the point d, since the RFG-open set $\{d\}$ is such that $d \in \{d\} \subset \{a,d\}$. However, the set $\{a,d\}$ is not a neighborhood of the point d, since no open set G exists such that $d \in G \subset \{a,d\}$.

Theorem 4.6. *If a subset N of a space X is RFG-open. Then N is a RFG-nbhd of each of its points.*

Proof. Suppose N is RFG-open. Let $x \in X$. We claim that N is RFG-nbhd of x. For A is a RFG-open set such that $x \in A \subset N$. Since x is an arbitrary point of N, it follows that N is a RFG-nbhd of each of its points. □

Remark 4.7. The converse of the above theorem is not true in general as seen from the following example.

Example 4.8. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{c,d\},\{a,c,d\}\}$. Then $RFGO(X) = \{X,\phi,\{a\},\{c\},\{d\},\{c,d\},\{a,c,d\}\}$. The set $\{a,d\}$ is RFG-nbhd of the point d, since the RFG-open set $\{d\}$ is such that $d \in \{d\} \subset \{a,d\}$. Also the set $\{a,d\}$ is a RFG-nbhd of the point a, since the RFG-open set $\{a\}$ is such that $a \in \{a\} \subset \{a,d\}$. (i.e.) $\{a,d\}$ is a RFG-nbhd of each of its points. However the set $\{a,d\}$ is not a RFG-open set in X.

Theorem 4.9. *Let X be a topological space. If F is a RFG-closed subset of X and $x \in F^c$. Then there exists a RFG-nbhd N of x such that $N \cap F = \phi$.*

Proof. Let F be RFG-closed subset of X and $x \in F^c$. Then F^c is RFG-open set of X. So by Theorem 5.4, F^c contains a RFG-nbhd of each of its points. Hence there exists a RFG-nbhd N of x such that $N \subset F^c$. That is $N \cap F = \phi$. □

Theorem 4.10. *Let X be a topological space and for each $x \in X$, let $RFG\text{-}N(x)$ be the collection of all RFG-nbhds of x. Then we have the following results.*

(i) $\forall\ x \in X,\ RFG\text{-}N(x) \neq \phi$.

(ii) $N \in RFG\text{-}N(x) \Rightarrow x \in N$.

(iii) $N \in RFG\text{-}N(x)$ *and* $N \subset M \Rightarrow M \in RFG\text{-}N(x)$.

(iv) $N \in RFG\text{-}N(x) \Rightarrow \exists\ M \in RFG\text{-}N(x)$ *such that* $M \subset N$ *and* $M \in RFG\text{-}N(y)$ *for every* $y \in M$.

Proof. (i) Since X is an RFG-open set, it is a RFG-nbhd of every $x \in X$. Hence $\exists$ atleast one RFG-nbhd(x) for each $x \in X$. Hence $RFG\text{-}N(x) \neq \phi$ for every $x \in X$.

(ii) If $N \in RFG\text{-}N(x)$. Then N is a RFG-nbhd of x. So by definition of RFG-nbhd, $x \in N$.

(iii) Let $N \in RFG\text{-}N(x)$ and $N \subset M$. Then there is an RFG-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is a RFG-nbhd of x.

Hence $M \in RFG\text{-}N(x)$.

(iv) If $N \in RFG\text{-}N(x)$. Then there exists an RFG-open set M such that $x \in M \subset N$. Since M is an RFG-open set, it is a RFG-nbhd of each of its points. Therefore, $M \in RFG\text{-}N(y)$ for every $y \in M$. □

5. Regular Feebly Generalized Interior (RFG-Interior) Operator

In this section, the notation of RFG-interior is defined and some of its basic properties are studied.

Definition 5.1. (i) Let A be a subset of (X,τ). A point $x \in A$ is said to be RFG-interior point of A if and only if A is RFG-neighborhood of x. The set of all RFG-interior points of A is called the RFG-interior of A and is denoted by $RFG\text{-}int(A)$.

(ii) Let (X,τ) be a topological space and $A \subset X$. Then $RFG\text{-}int(A)$ is the union all RFG-open sets contained in A.

Theorem 5.2. *Let A is subset of (X,τ). Then $RFG\text{-}int(A) = \cup\{G : G$ is RFG-open, $G \subset A\}$.*

Proof. Let A be a subset of (X,τ) and $x \in RFG\text{-}int(A)$
$\Leftrightarrow$ x is a RFG-interior point of A
$\Leftrightarrow$ A is a RFG-nbhd of point x
$\Leftrightarrow$ there exists an RFG-open set G such that $x \in G \subset A$
$\Leftrightarrow$ $x \in \cup\{G : G$ is RFG-open, $G \subset A\}$
Hence $RFG\text{-}int(A) = \cup\{G : G$ is RFG-open, $G \subset A\}$. □

Theorem 5.3. *Let A and B are subsets of (X,τ). Then*

(i) *$RFG\text{-}int(\phi) = \phi$ and $RFG\text{-}int(X) = X$.*
(ii) *$RFG\text{-}int(A) \subset A$.*
(iii) *If B is any RFG-open set contained in A. Then $B \subset RFG\text{-}int(A)$.*
(iv) *If $A \subset B$. Then $RFG\text{-}int(A) \subset RFG\text{-}int(B)$.*
(v) *$RFG\text{-}int(RFG\text{-}int(A)) = RFG\text{-}int(A)$.*

Proof. (i) Obvious

(ii) Let $x \in RFG\text{-}int(A) \Rightarrow x$ is a RFG-interior point of $A \Rightarrow A$ is a RFG-nbhd of $x \Rightarrow x \in A$. Thus $x \in RFG\text{-}int(A) \Rightarrow x \in A$. Hence $RFG\text{-}int(A) \subset A$.

(iii) Let B be any RFG-open set such that $B \subset A$. Let $x \in B$. Since B is an RFG-open set contained in A, x is an RFG-interior point of A. (i.e.) $x \in RFG\text{-}int(A)$.

(iv) Let A and B are subsets of X such that $A \subset B$. Let $x \in RFG\text{-}int(A)$. Then x is an RFG-interior point of A and so A is a RFG-nbhd of x. Since $A \subset B$, B is also a RFG-nbhd of x. This implies that $x \in RFG\text{-}int(B)$. Thus we have, $x \in RFG\text{-}int(A)$. Hence $B \subset RFG\text{-}int(A)$.

$\Rightarrow x \in RFG\text{-}int(B)$. Hence $RFG\text{-}int(A) \subset RFG\text{-}int(B)$.

(v) Since $RFG\text{-}int(A)$ is a RFG-open set in X, it follows that $RFG\text{-}int(RFG\text{-}int(A)) = RFG\text{-}int(A)$. □

Theorem 5.4. *If a subset A of the space X is RFG-open. Then $RFG\text{-}int(A) = A$.*

Proof. Let A be a RFG-open subset of X and we know that $RFG\text{-}int(A) \subset A$. Since A is RFG-open set contained in A and from the Theorem 5.3(iii), $A \subset RFG\text{-}int(A)$ and hence we get $RFG\text{-}int(A) = A$. □

The converse of the above theorem need not be true as seen in the following example.

Example 5.5. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then $RFGO(X) = \{X,\phi,\{a\},\{b\},\{c\},\{b,c\},\{a,b,c\}\}$.

Note that $RFG\text{-}int(\{a,c\}) = \{\{a\} \cup \{c\} \cup \phi\} = \{a,c\}$. But $\{a,c\}$ is not a RFG-open set in X.

Theorem 5.6. *If A and B are subsets of X. Then $RFG\text{-}int(A) \cup RFG\text{-}int(B) \subset RFG\text{-}int(A \cup B)$.*

Proof. Let A and B be subsets of X. Clearly, $A \subset A \cup B$ and $B \subset A \cup B$. By Theorem 5.3(iv), $RFG\text{-}int(A) \subset RFG\text{-}int(A \cup B)$ and $RFG\text{-}int(B) \subset RFG\text{-}int(A \cup B)$. This implies that $RFG\text{-}int(A) \cup RFG\text{-}int(B) \subset RFG\text{-}int(A \cup B)$. □

Theorem 5.7. *If A and B are subsets of X. Then $RFG\text{-}int(A \cap B) \subset RFG\text{-}int(A) \cap RFG\text{-}int(B)$.*

Proof. Let A and B be subsets of X. Clearly, $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem 5.3(iv), $RFG\text{-}int(A \cap B) \subset RFG\text{-}int(A)$ and $RFG\text{-}int(A \cap B) \subset RFG\text{-}int(B)$.
Hence $RFG\text{-}int(A \cap B) \subset RFG\text{-}int(A) \cap RFG\text{-}int(B)$. □

Theorem 5.8. *If A is a subset of X. Then $int(A) \subset RFG\text{-}int(A)$.*

Proof. Let A be a subset of X. $x \in int(A) \Longrightarrow x \in \cup\{G : G$ is open, $G \subset A\}$.
$\Rightarrow$ There exists an open set G such that $x \in G \subset A$.
$\Rightarrow$ There exists an RFG-open set G such that $x \in G \subset A$, as every open set is an RFG-openset in X.
$\Rightarrow x \in \cup\{G : G$ is RFG-open, $G \subset A\} \Rightarrow x \in RFG\text{-}int(A)$.
Thus $x \in int(A) \Rightarrow x \in RFG\text{-}int(A)$. Hence $int(A) \subset RFG\text{-}int(A)$. □

Remark 5.9. Containment relation in the above Theorem 5.8 may be proper as seen from the following example.

Example 5.10. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Then $RFGO(X) = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$. Let $A = \{a,b,d\}$.
Now $RFG\text{-}int(A) = \{a,b,d\}$ and $int(A) = \{a,b\}$.

6. Regular Feebly Generalized Closure (RFG-Closure) Operator

Now we introduce the notation of RFG-closure in topological spaces by using the concept of RFG-closed sets and obtain some of their properties.

Definition 6.1. Let A be a subset of a space (X,τ). We define the RFG-closure of A to be a intersection of all RFG-closed sets containing A. In symbol, we have

$$RFG\text{-}cl(A) = \cap\{F : A \subset F \in RFGC(X)\}.$$

Theorem 6.2. *Let A and B are subsets of (X,τ). Then*

(i) $RFG\text{-}cl(\phi) = \phi$ *and* $RFG\text{-}cl(X) = X$

(ii) $A \subset RFG\text{-}cl(A)$.

(iii) *If B is any RFG-closed set containing A. Then $RFG\text{-}cl(A) \subset B$.*

(iv) *If* $A \subset B$*. Then* RFG-$cl(A) \subset RFG$-$cl(B)$.

(v) RFG-$cl(RFG$-$cl(A)) = RFG$-$cl(A)$.

Proof. (i) Obvious.

(ii) By the definition of RFG-closure of A, it is obvious that $A \subset RFG$-$cl(A)$.

(iii) Let B be any RFG-closed set containing A. Since RFG-$cl(A)$ is the intersection of all RFG-closed set containing A, RFG-$cl(A)$ is contained in every RFG-closed set containing A. Hence in particular RFG-$cl(A) \subset B$.

(iv) Let A and B be subsets of X such that $A \subset B$. By the definition of RFG-closure, RFG-$cl(B) = \cap\{F : B \subset F \in RFGC(X)\}$. If $B \subset F \in RFGC(X)$. Then RFG-$cl(B) \subset F$. Since $A \subset B$, $A \subset B \subset F \in RFGC(X)$. We have RFG-$cl(A) \subset F$. Therefore RFG-$cl(A) \subset \cap\{F : B \subset F \in RFGC(X)\} = RFG$-$cl(B)$. (i.e.) RFG-$cl(A) \subset RFG$-$cl(B)$.

(v) Since RFG-$cl(A)$ is a RFG-closed set in X, it follows that RFG-$cl(RFG$-$cl(A)) = RFG$-$cl(A)$. □

Theorem 6.3. *If a subset A of the space X is RFG-closed, Then* RFG-$cl(A) = A$.

Proof. Let A be a RFG-closed subset of X and we know that $A \subset RFG$-$cl(A)$. Since A is RFG-closed set containing A and from the Theorem 6.2(iii), RFG-$cl(A) \subset A$. Hence we get RFG-$cl(A) = A$. □

The converse of the above theorem need not be true as seen in the following example.

Example 6.4. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X, \phi, \{a\}, \{c,d\}, \{a,c,d\}\}$.
Then $RFGC(X) = \{X, \phi, \{b\}, \{a,b\}, \{a,b,c\}, \{b,c,d\}, \{a,b,d\}\}$.
Now RFG-$cl(\{b,c\}) = \{\{a,b,c\} \cap \{b,c,d\} \cap X\} = \{b,c\}$. But $\{b,c\}$ is not a RFG-closed subset in X.

Theorem 6.5. *If A and B are subsets of X, Then* RFG-$cl(A) \cup RFG$-$cl(B) \subset RFG$-$cl(A \cup B)$.

Proof. Let A and B be subsets of a space X. Clearly $A \subset A \cup B$ and $B \subset A \cup B$.
By Theorem 6.2(iv), RFG-$cl(A) \subset RFG$-$cl(A \cup B)$ and RFG-$cl(B) \subset RFG$-$cl(A \cup B)$.
This implies that RFG-$cl(A) \cup RFG$-$cl(B) \subset RFG$-$cl(A \cup B)$. □

Theorem 6.6. *If A and B are subsets of X. Then* RFG-$cl(A \cap B) \subset RFG$-$cl(A) \cap RFG$-$cl(B)$.

Proof. Let A and B be subsets of X. Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem 6.2(iv), RFG-$cl(A \cap B) \subset RFG$-$cl(A)$ and RFG-$cl(A \cap B) \subset RFG$-$cl(B)$.
Hence RFG-$cl(A \cap B) \subset RFG$-$cl(A) \cap RFG$-$cl(B)$. □

Theorem 6.7. *Let A be a subset of X and* $x \in X$*. Then* $x \in RFG$-$cl(A)$ *if and only if* $V \cap A \neq \emptyset$ *for every RFG-open set V containing x.*

Proof. Let $x \in X$ and $x \in RFG\text{-}cl(A)$. To prove that $V \cap A \neq \emptyset$ for every RFG-open set V containing x. We shall prove the theorem by contradiction. Suppose there exists a RFG-open set V containing x such that $V \cap A = \emptyset$. Then $A \subset X - V$ and $X - V$ is RFG-closed. We have $RFG\text{-}cl(A) \subset X - V$. This shows that $x \notin RFG\text{-}cl(A)$ which is a contradiction.
Hence $V \cap A \neq \emptyset$ for every RFG-open set V containing x.
Conversely, let $V \cap A \neq \emptyset$ for every RFG-open set V containing x. To prove that $x \in RFG\text{-}cl(A)$. We shall prove the result by contradiction. Suppose $x \notin RFG\text{-}cl(A)$. Then there exists a RFG-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ and $X - F$ is RFG-open. Also $(X - F) \cap A = \emptyset$ which is a contradiction. Hence $x \in RFG\text{-}cl(A)$. □

Theorem 6.8. *If A is a subset of X. Then $cl(A) \subset RFG\text{-}cl(A)$.*

Proof. Let A be a subset of X. By definition of closure, $cl(A) = \cap\{F \subset X : A \subset F \in C(X)\}$.
If $A \subset F \in C(X)$. Then $A \subset F \in RFGC(X)$, be cause every closed set is RFG-closed. That is $RFG\text{-}cl(A) \subset F$. Therefore $RFG\text{-}cl(A) \subset \cap\{F \subset X : A \subset F \in C(X)\} = cl(A)$.
Hence $RFG\text{-}cl(A) \subset cl(A)$. □

Remark 6.9. Containment relation in the above Theorem 6.8 may be proper as seen from the following example.

Example 6.10. Let $X = \{a,b,c,d\}$ with topology $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Then $RFGC(X) = \{X,\phi,\{c\},\{d\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$. Let $A = \{c\}$. Now $RFG\text{-}cl(A) = \{c\}$ and $cl(A) = \{c,d\}$. It follows that $RFG\text{-}cl(A) \subset cl(A)$ and $RFG\text{-}cl(A) \neq cl(A)$.

7. Conclusion

In this paper, we have focused on Regular Feebly Generalized closed (briefly RFG-closed) sets in topological spaces which lies be tween closed sets and feebly closed sets. This new class of set has more different properties which can be extended to different topological spaces. In future it is useful to extend some more research works in different topological spaces.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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